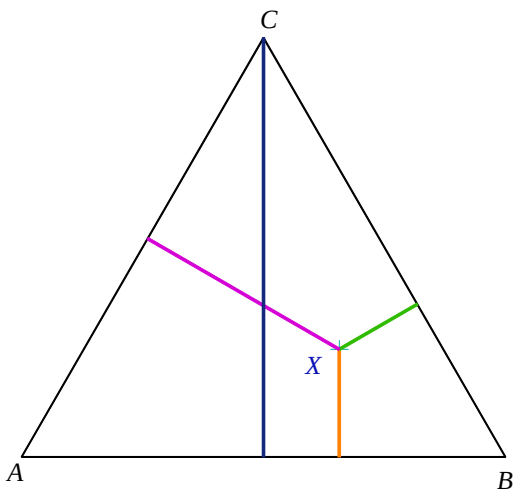
Důkazy takřka beze slov

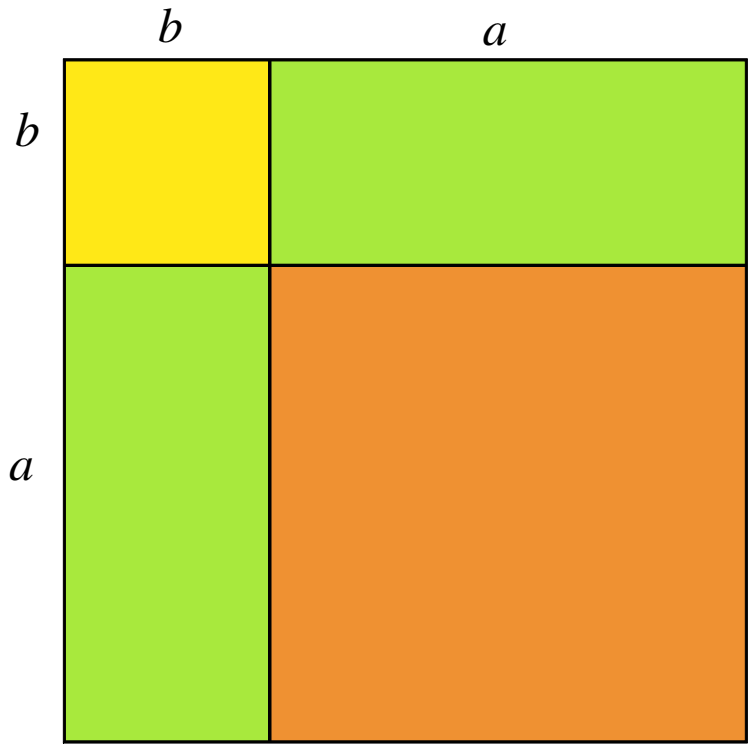
Martina Škorpilová

Cesty k matematice, Praha, 22.–23. 9. 2022



Druhá mocnina součtu

Pro kladná reálná čísla a, b platí $(a+b)^2 = a^2 + 2ab + b^2$.



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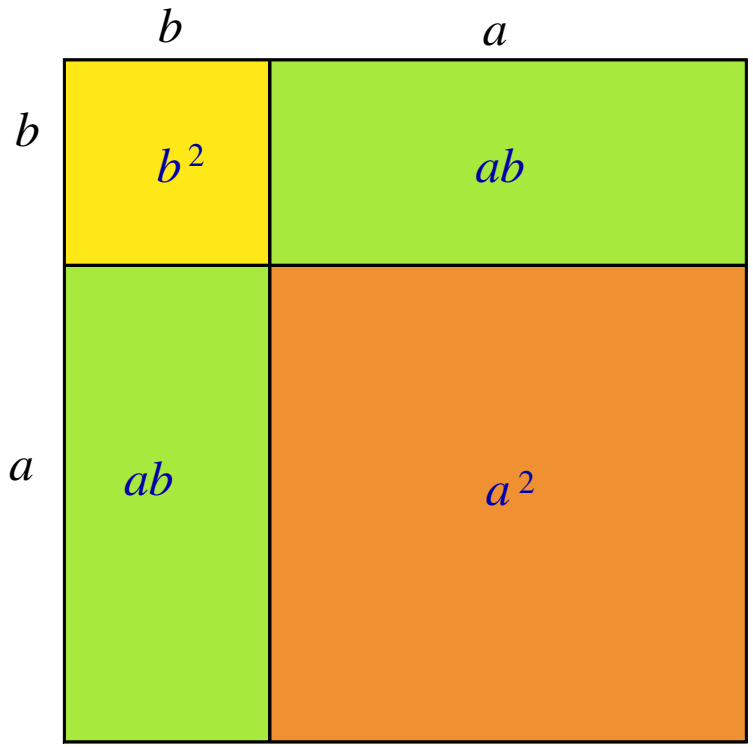
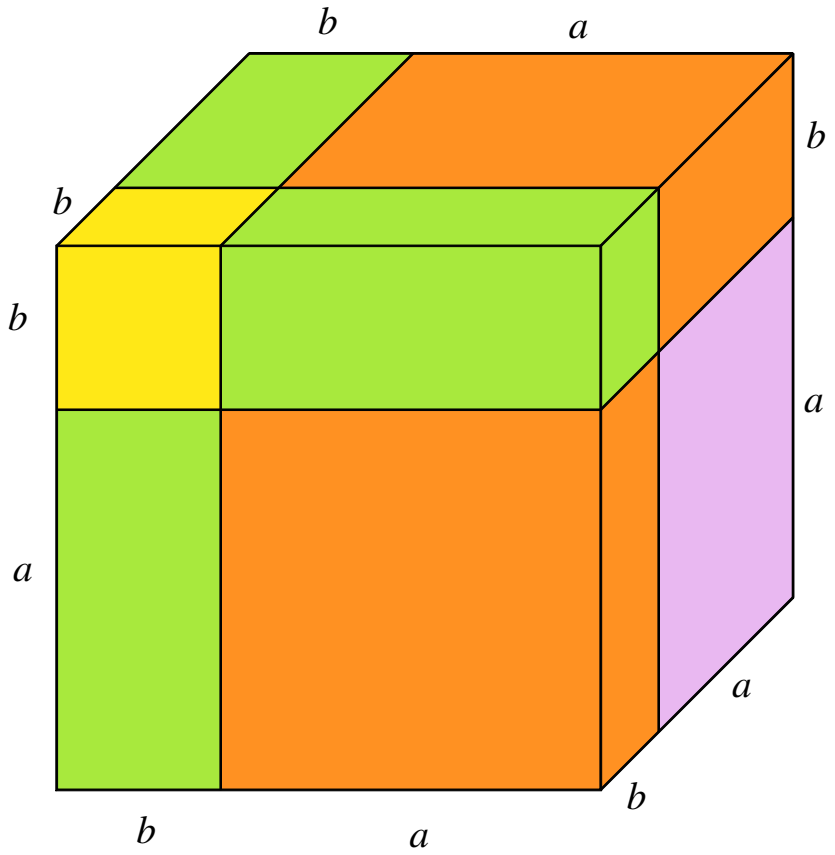


Třetí mocnina součtu

Pro kladná reálná čísla a, b platí $(a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$.



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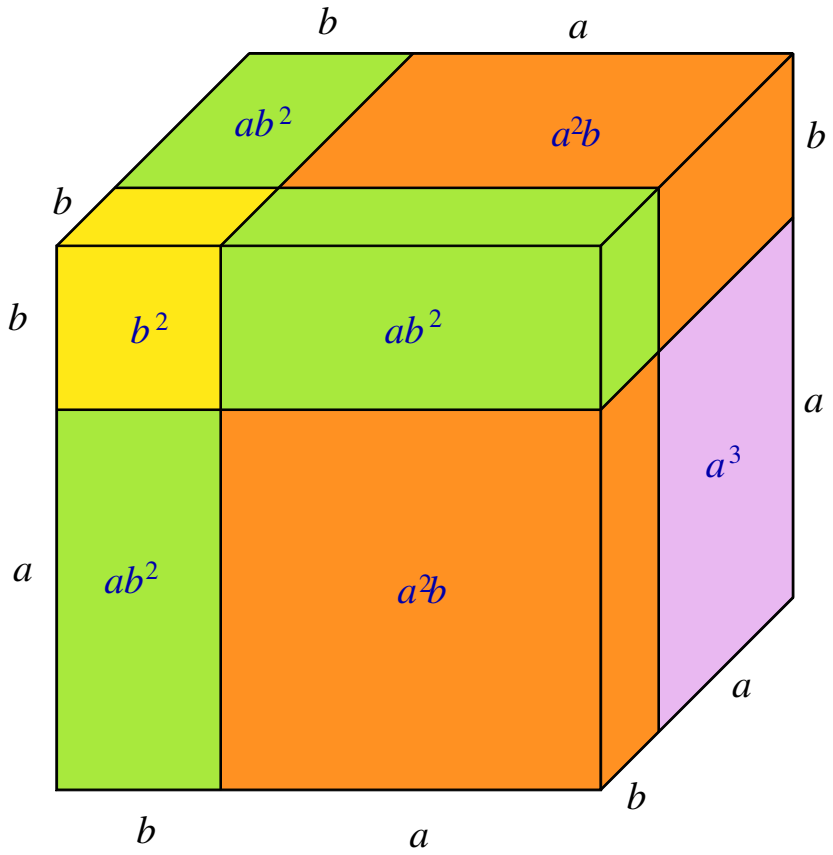
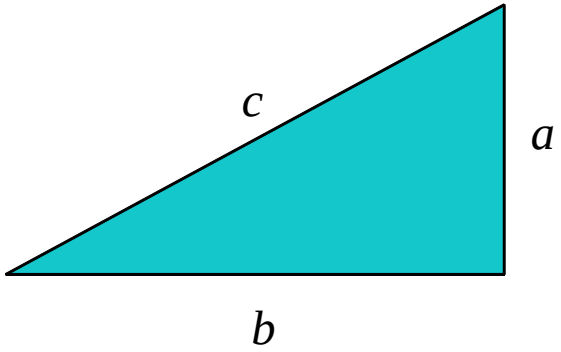


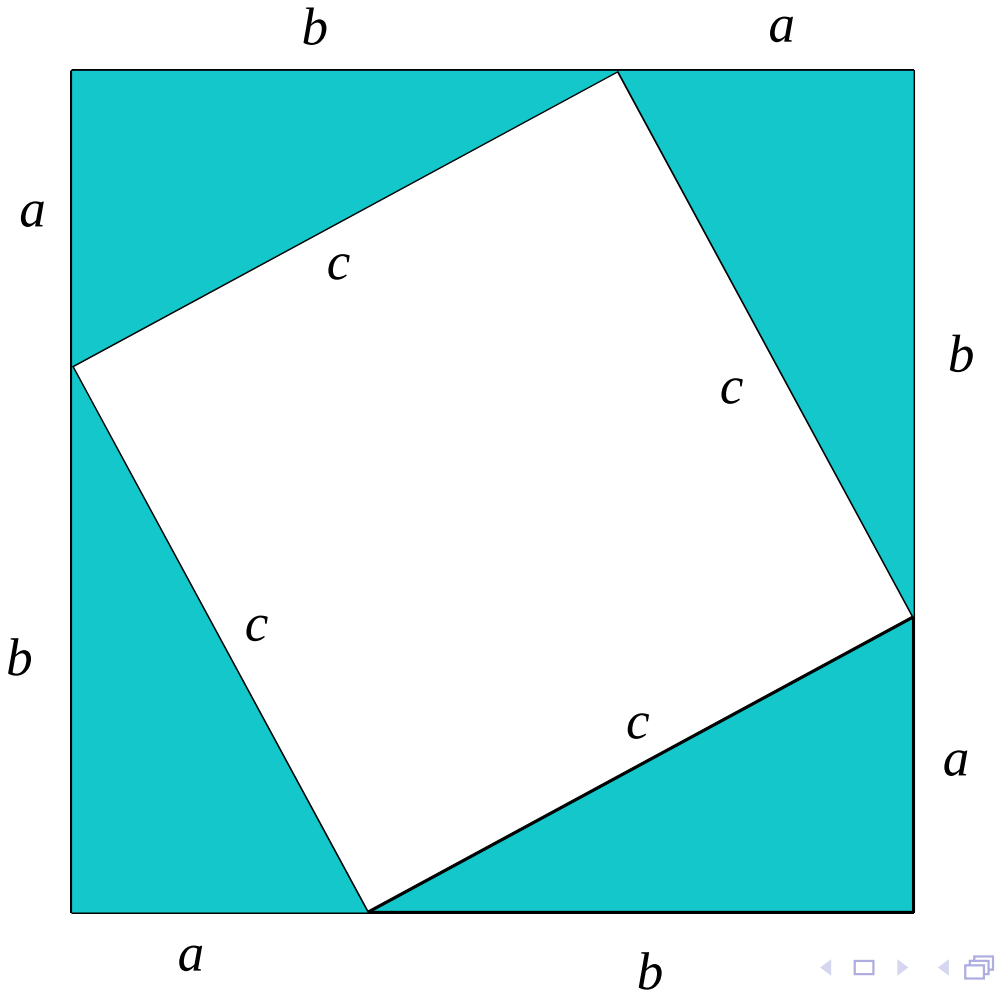
Pýthagorova věta

Pro pravoúhlý trojúhelník s odvěsnami a, b a přeponou c platí

$$a^2 + b^2 = c^2.$$



Pýthagorova věta



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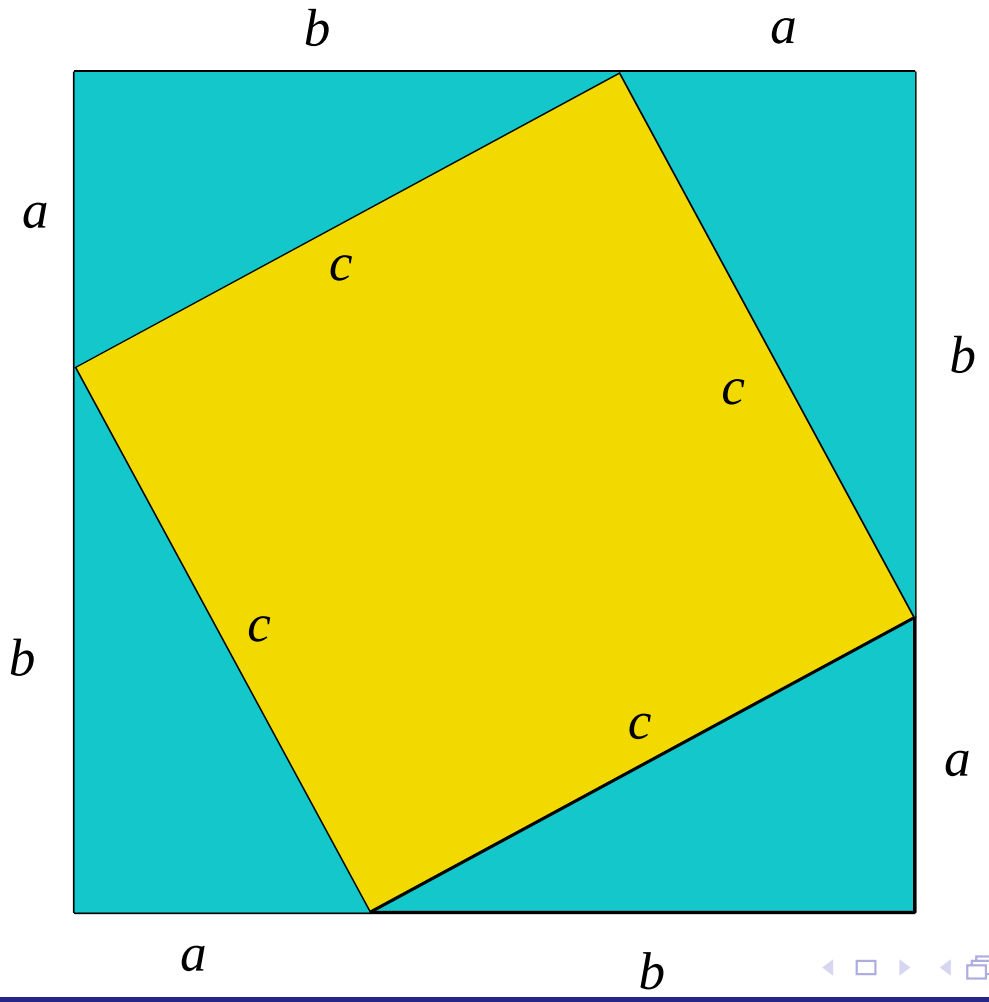
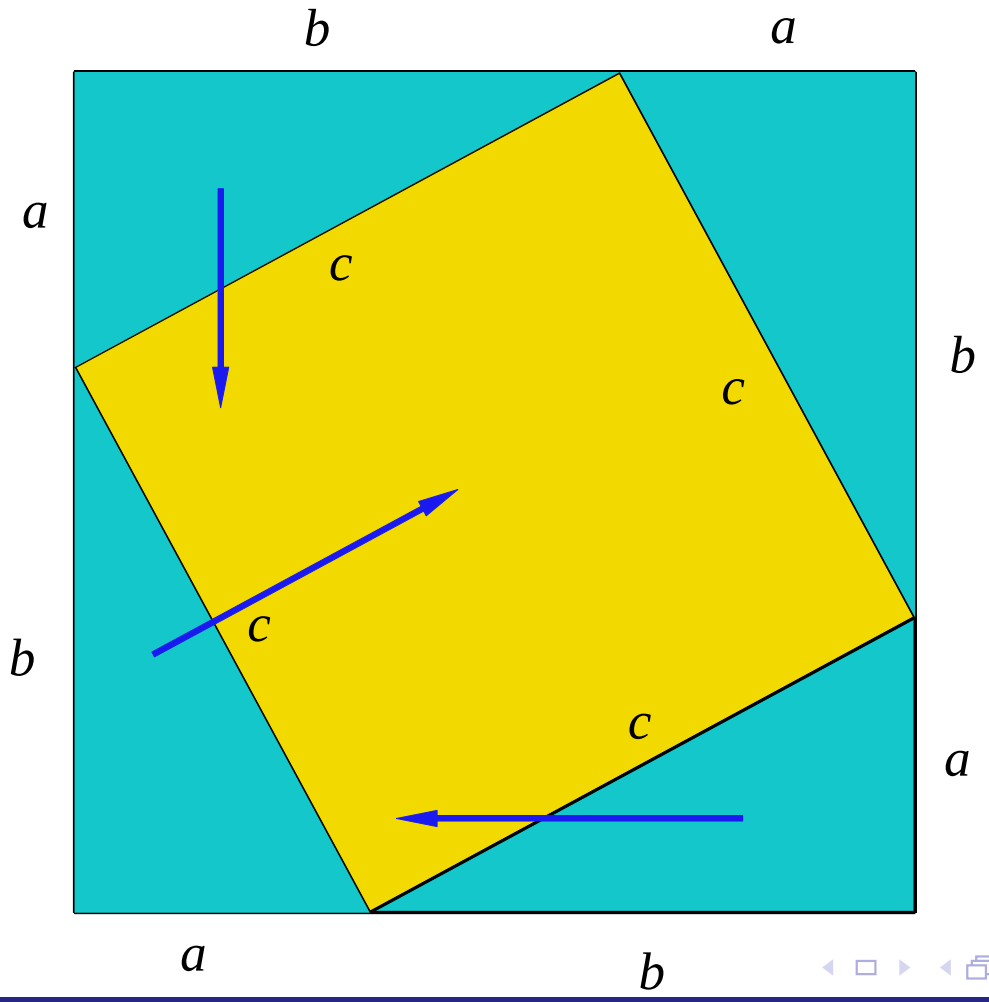
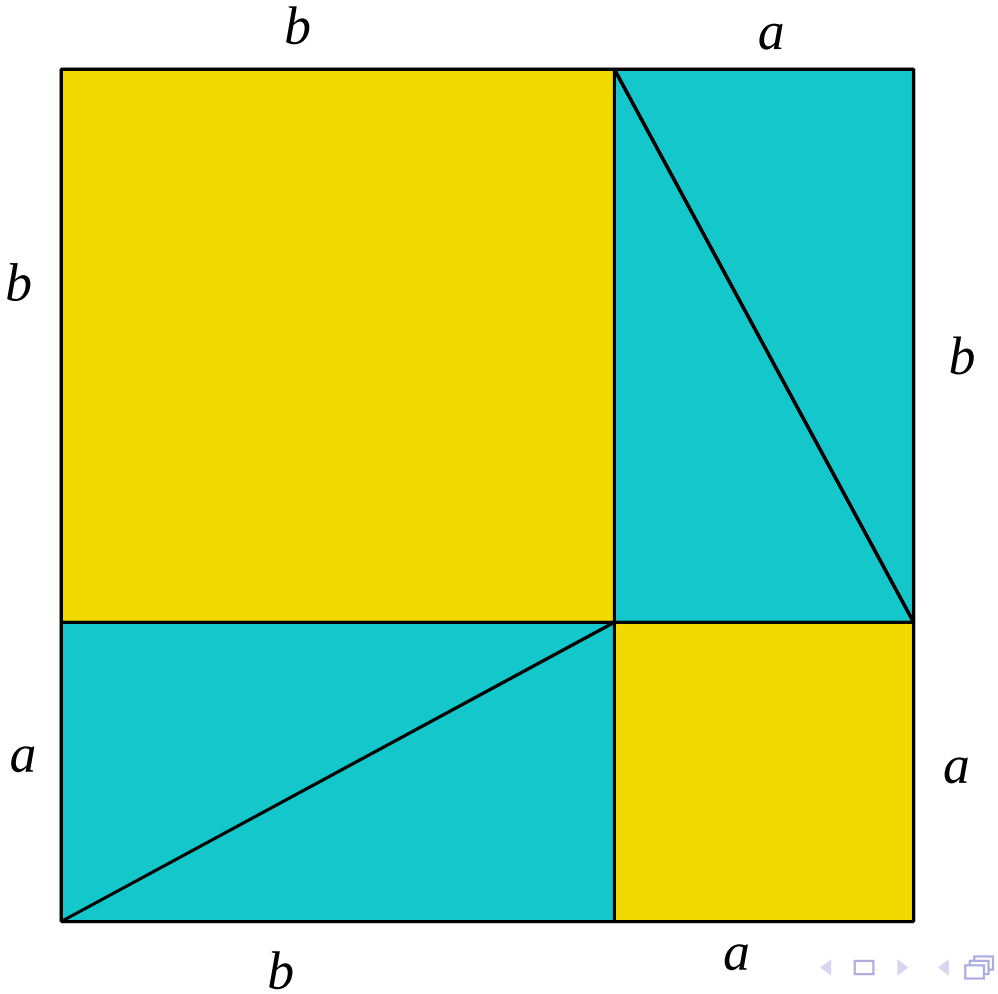


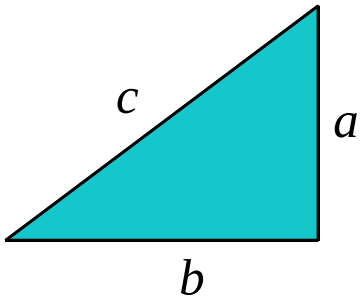
Pýthagorova věta



Pýthagorova věta



Pýthagorova věta – 2. způsob



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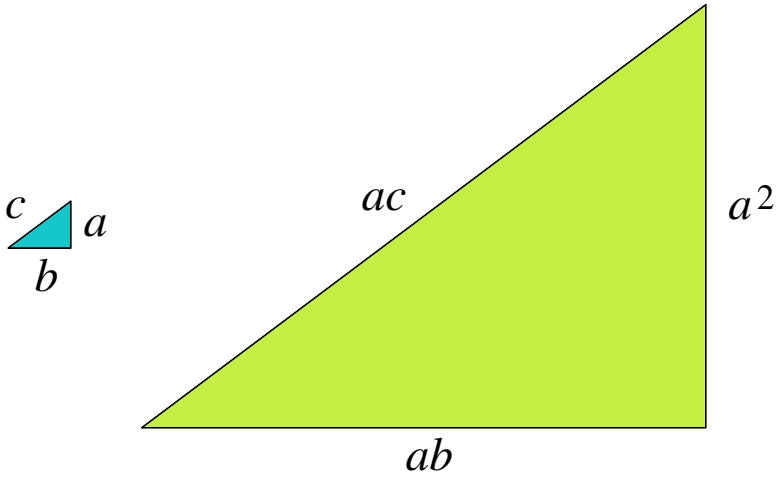

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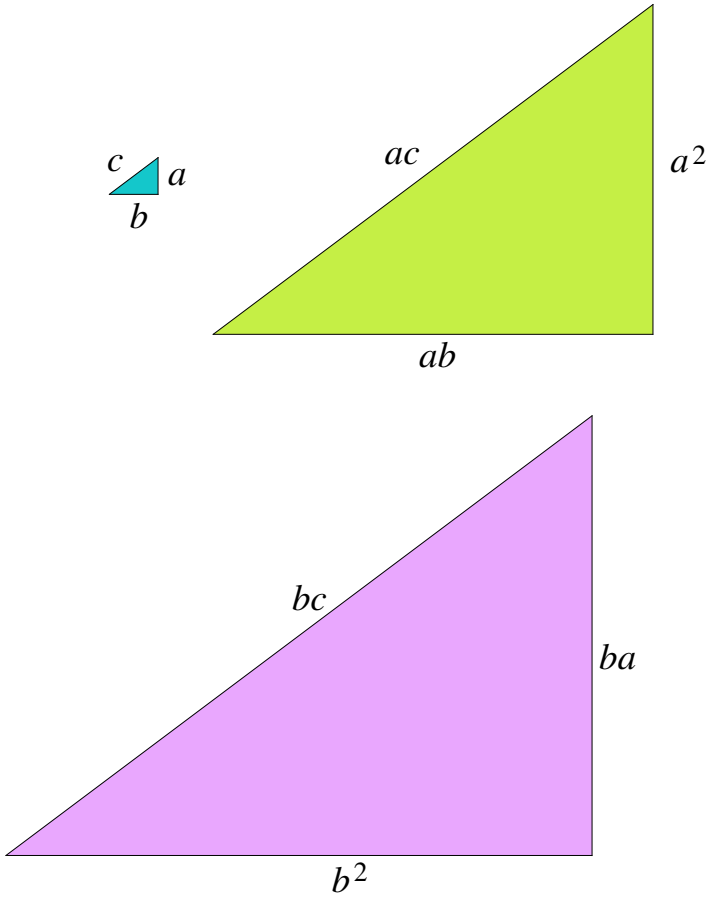

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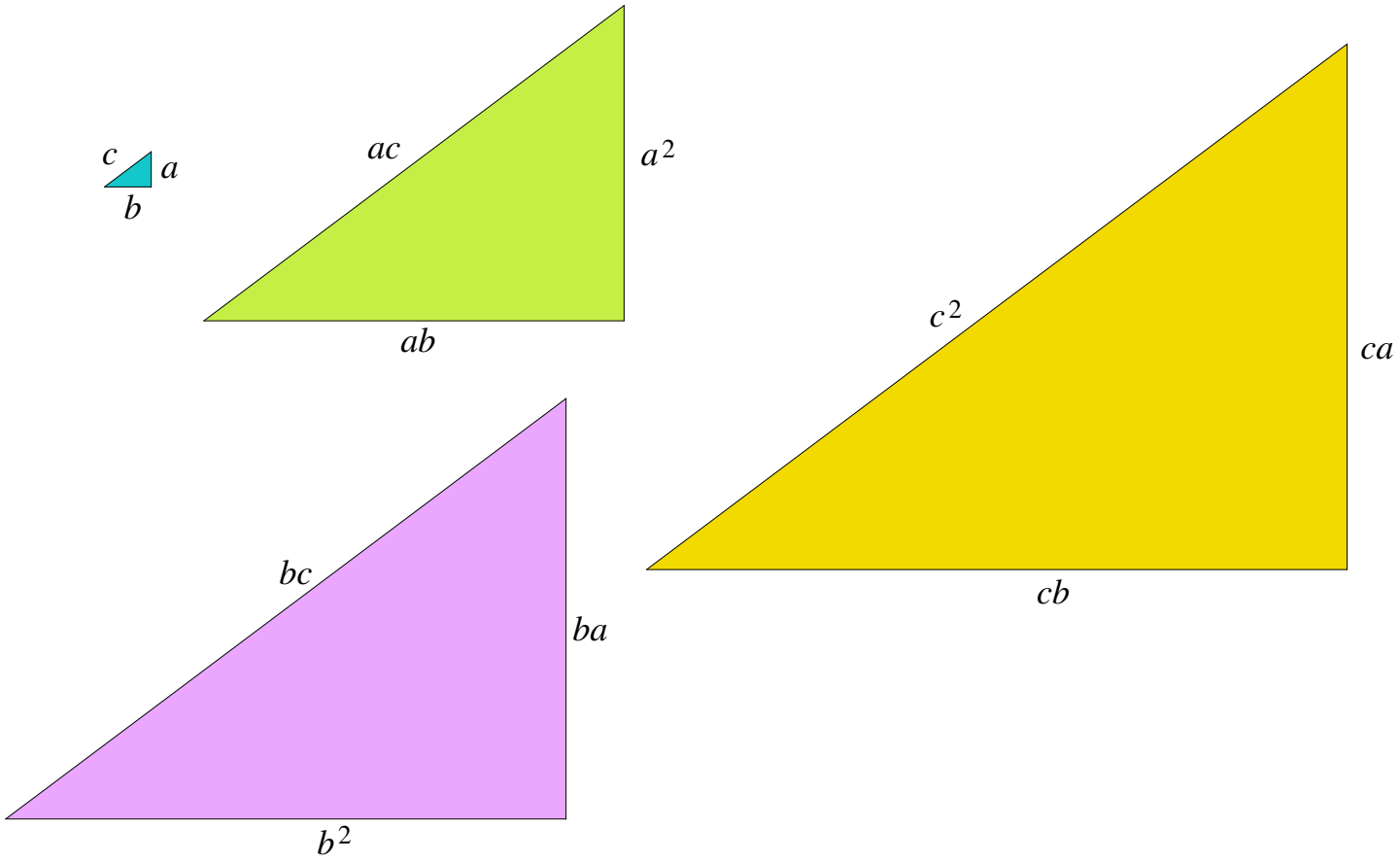

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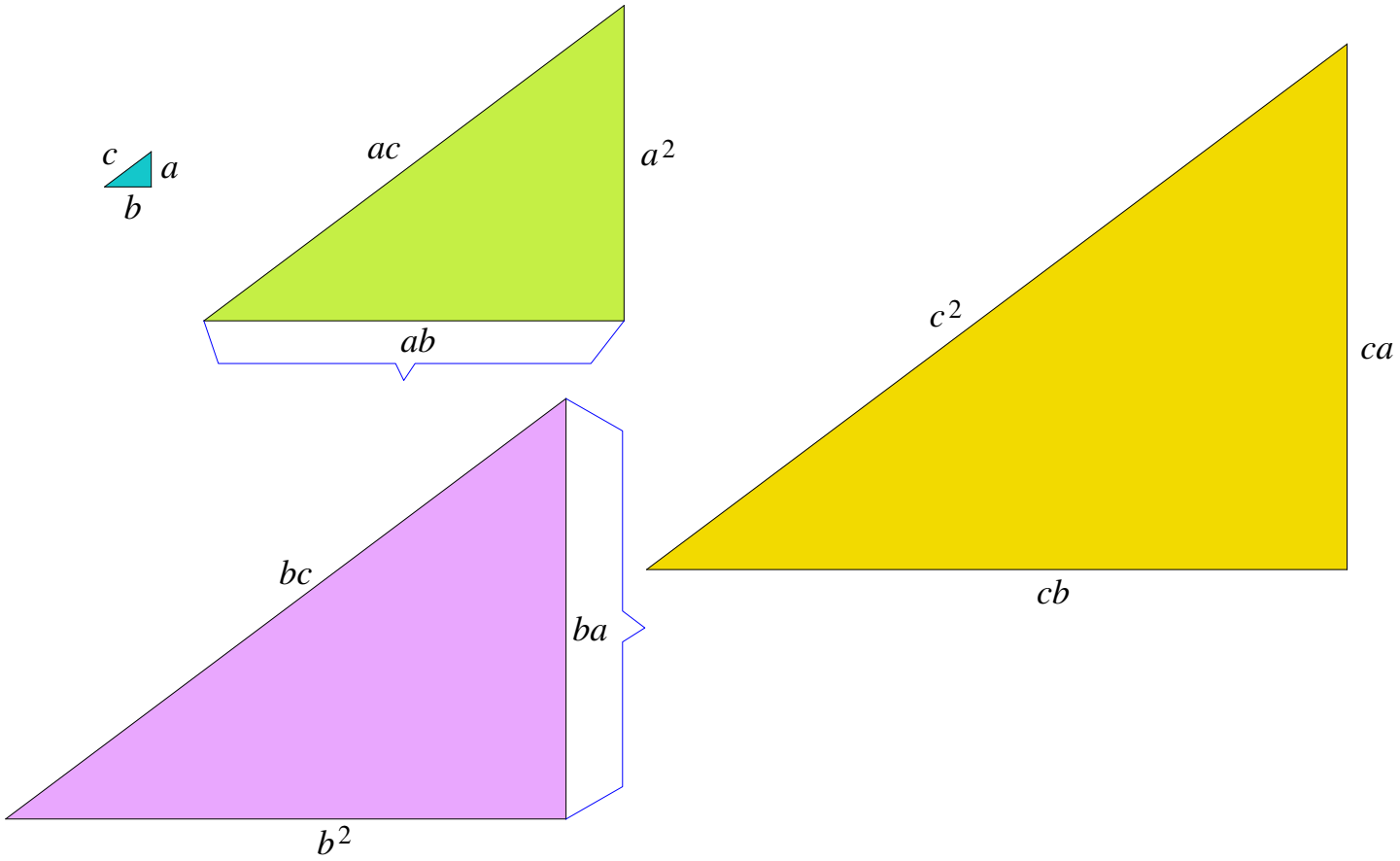

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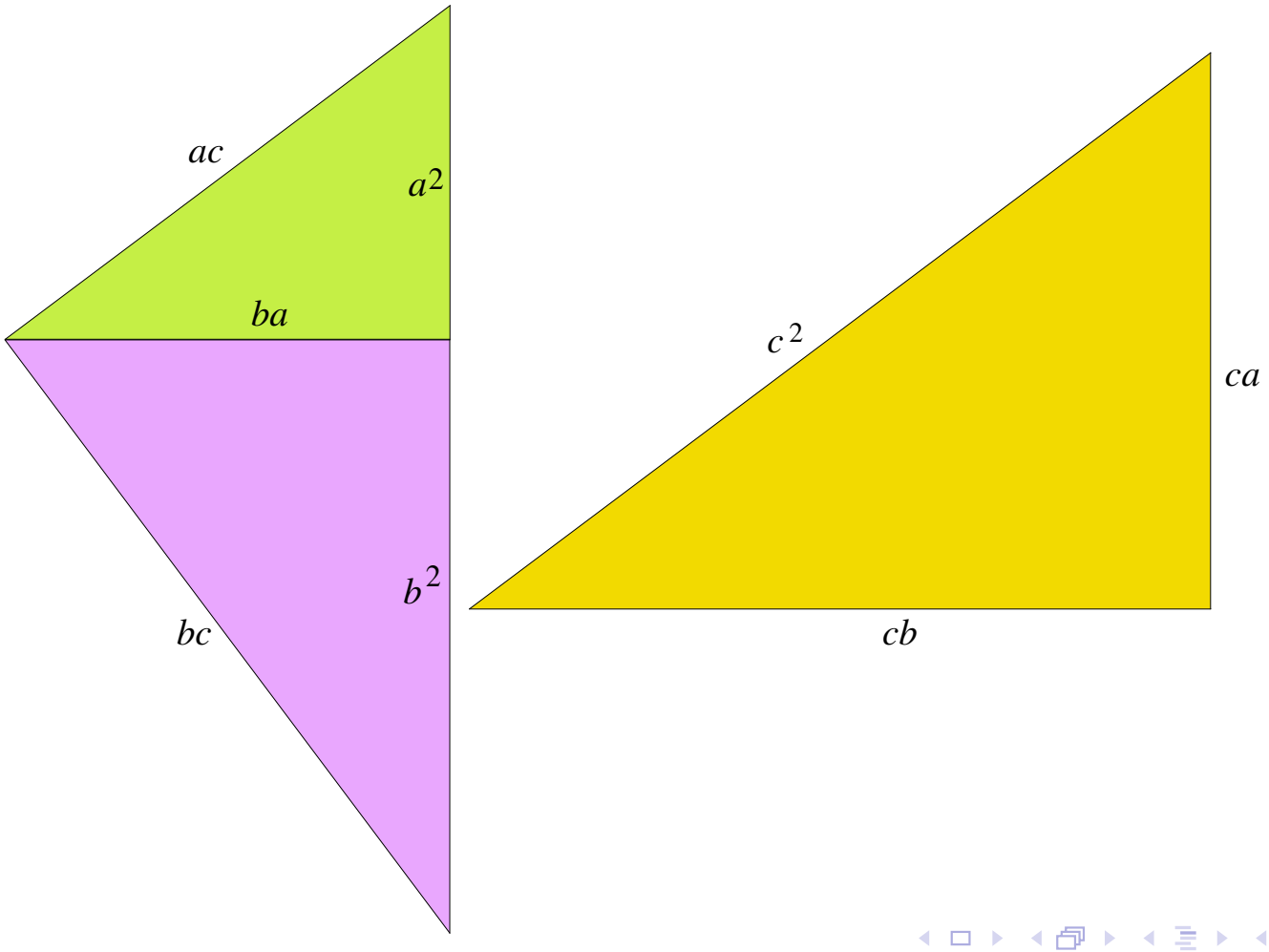

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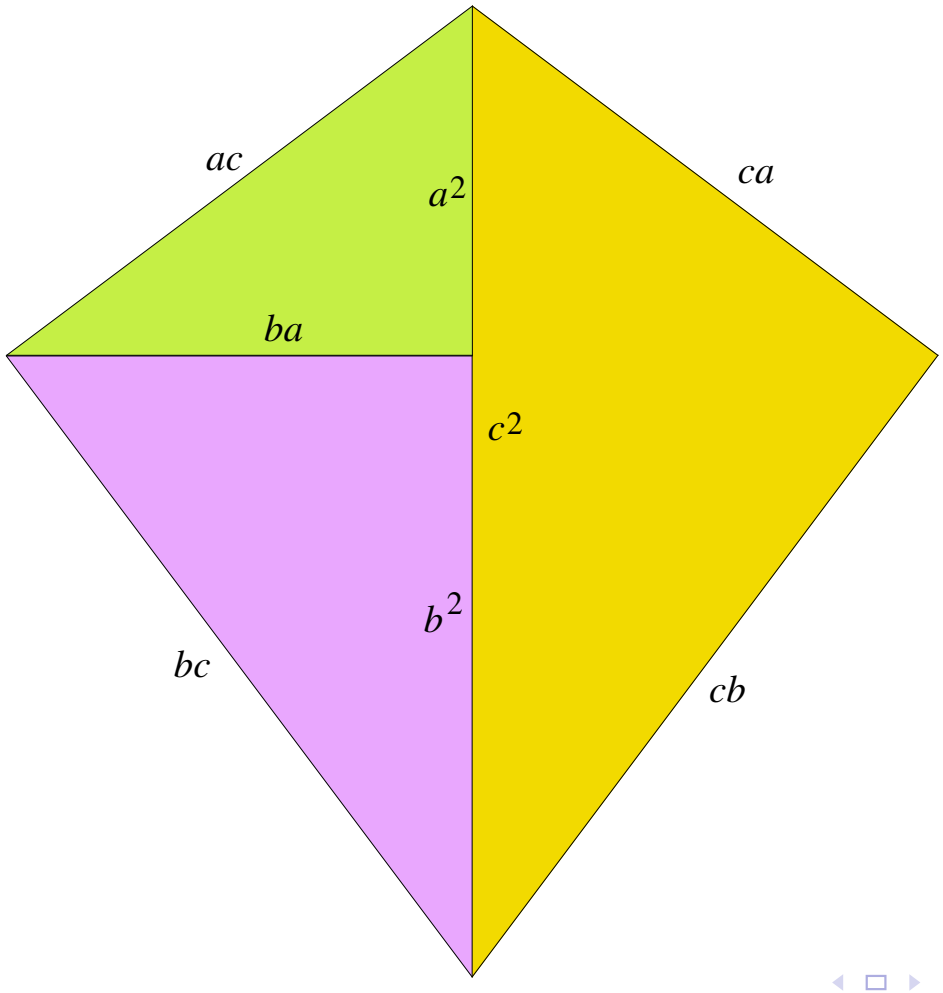
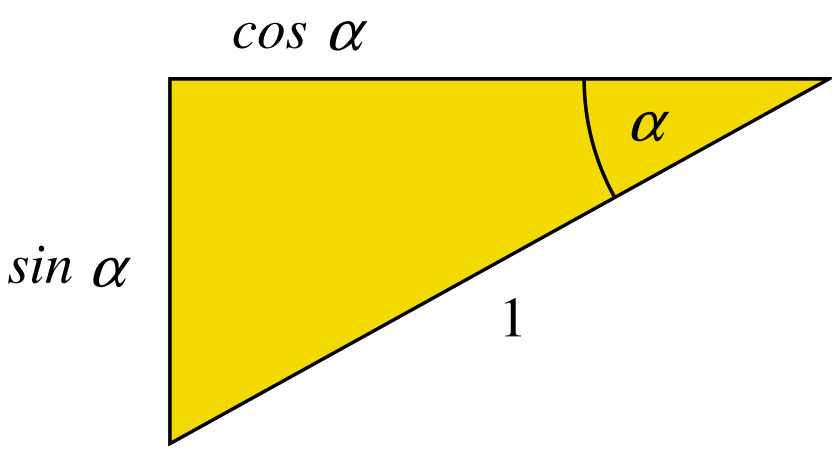


Součtový vzorec

$$\sin(\alpha+\beta) = \sin\alpha \cdot \cos\beta + \cos\alpha \cdot \sin\beta$$



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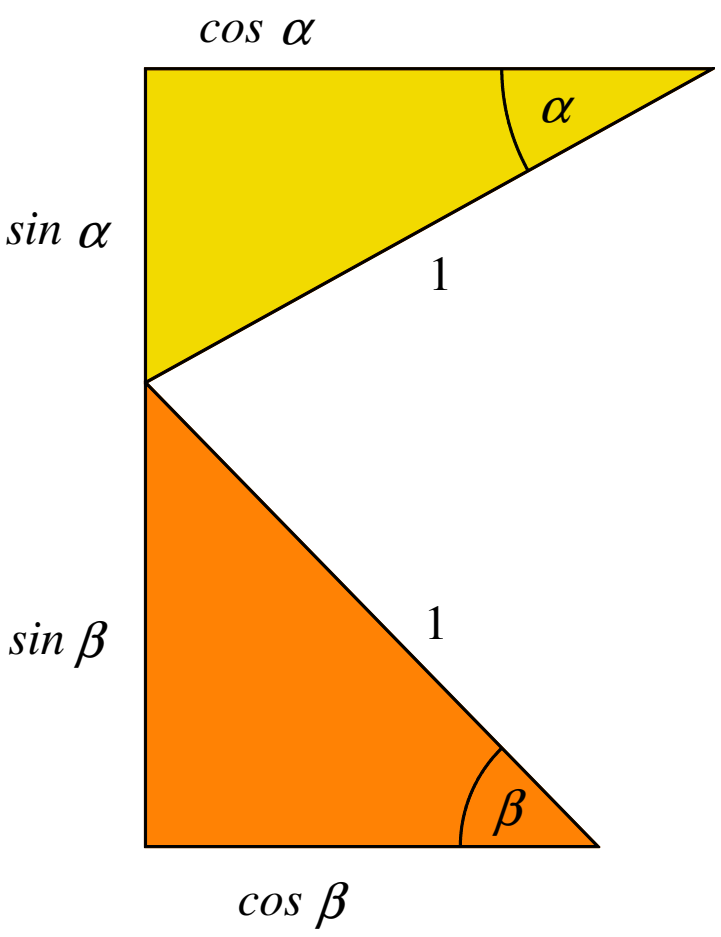

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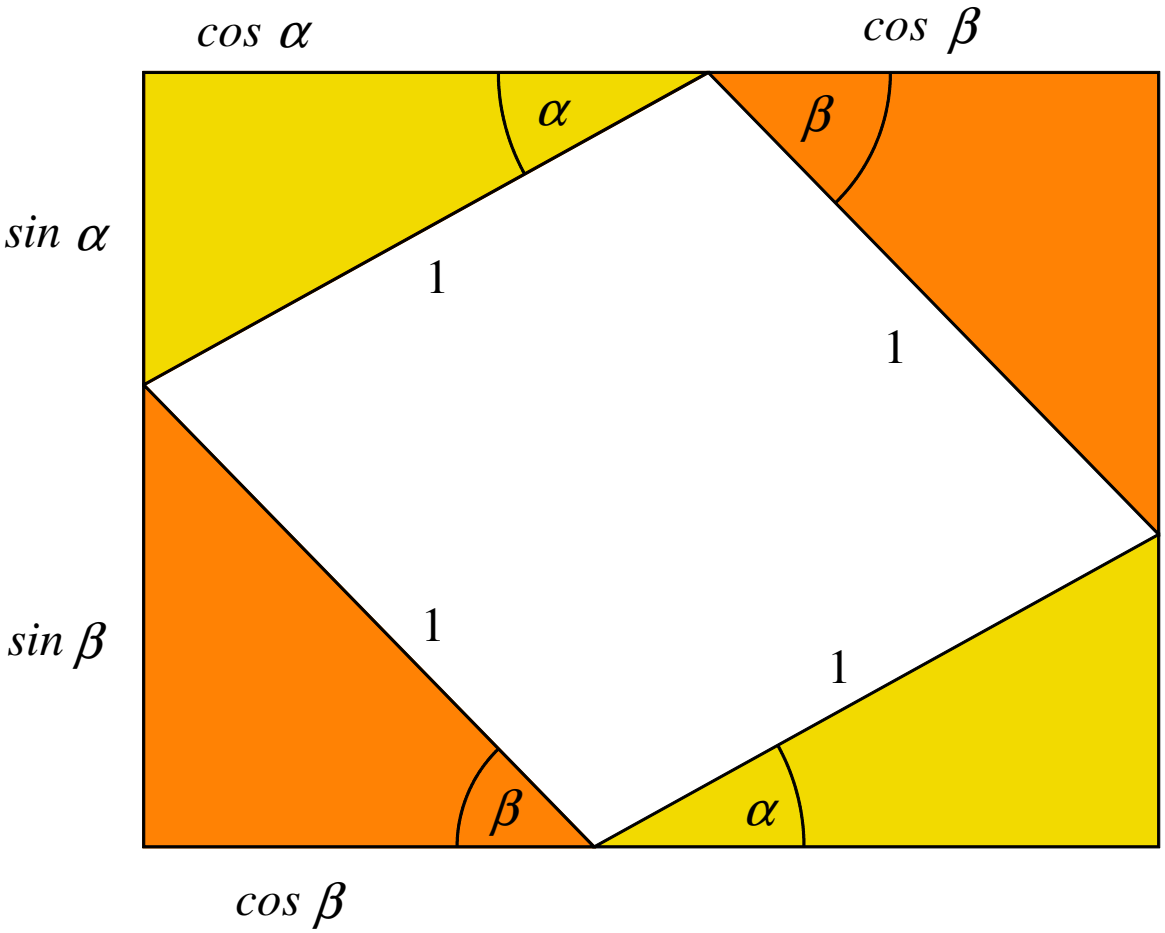

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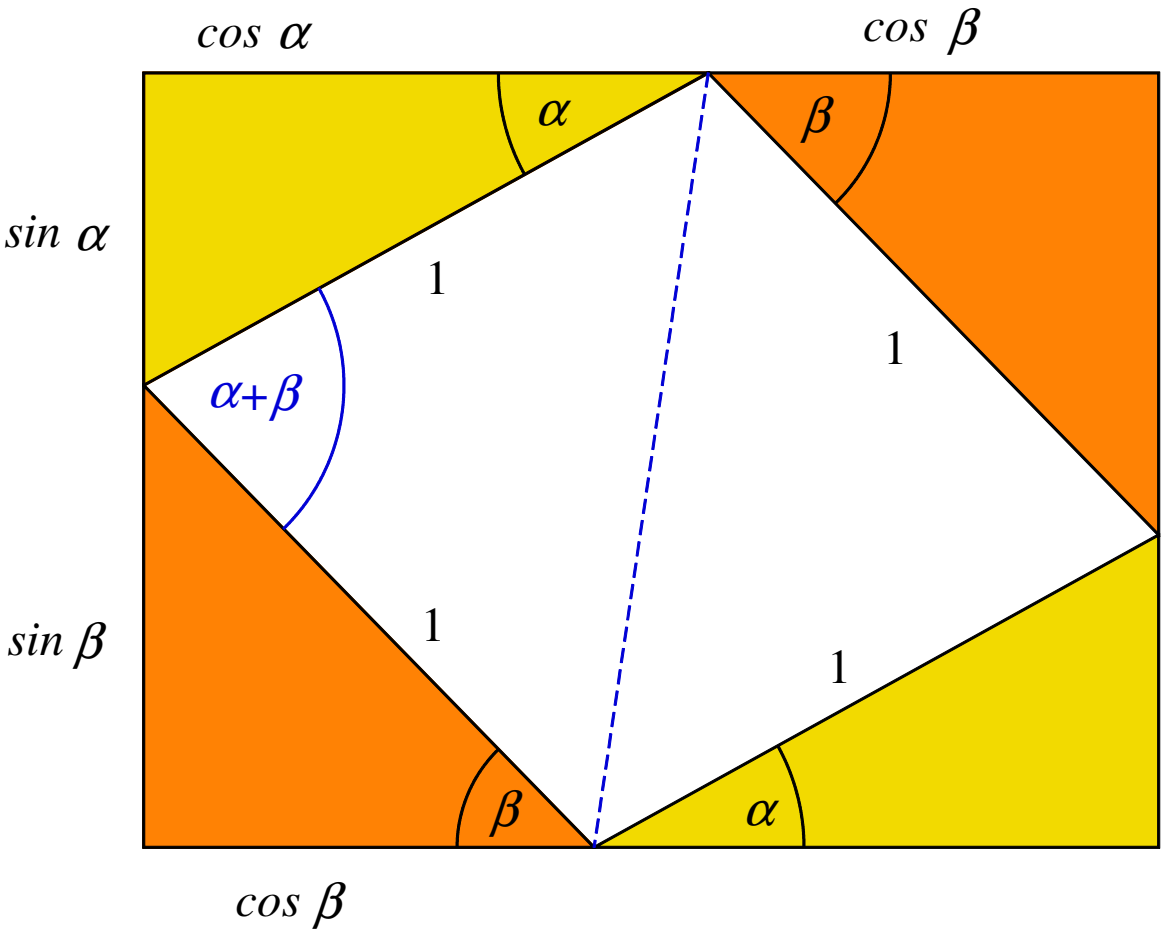

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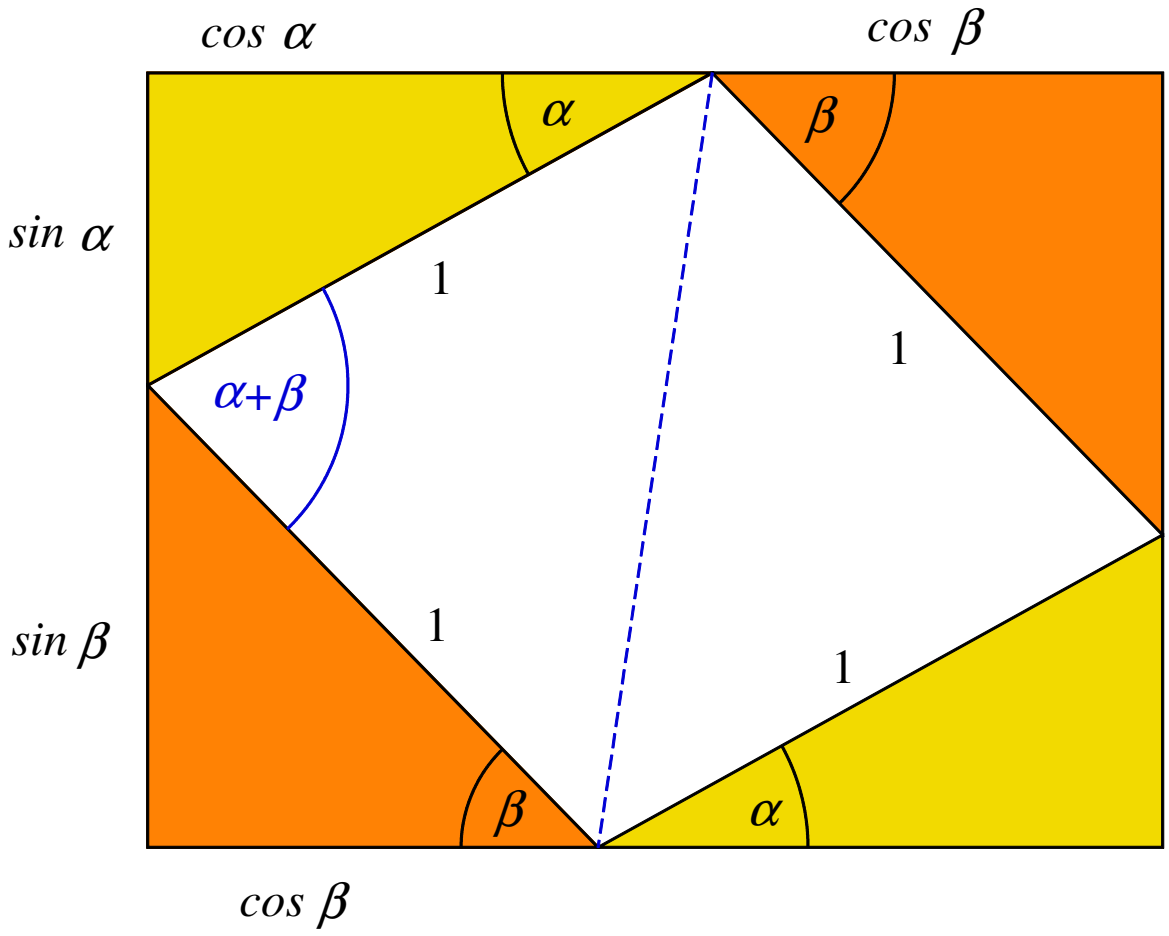
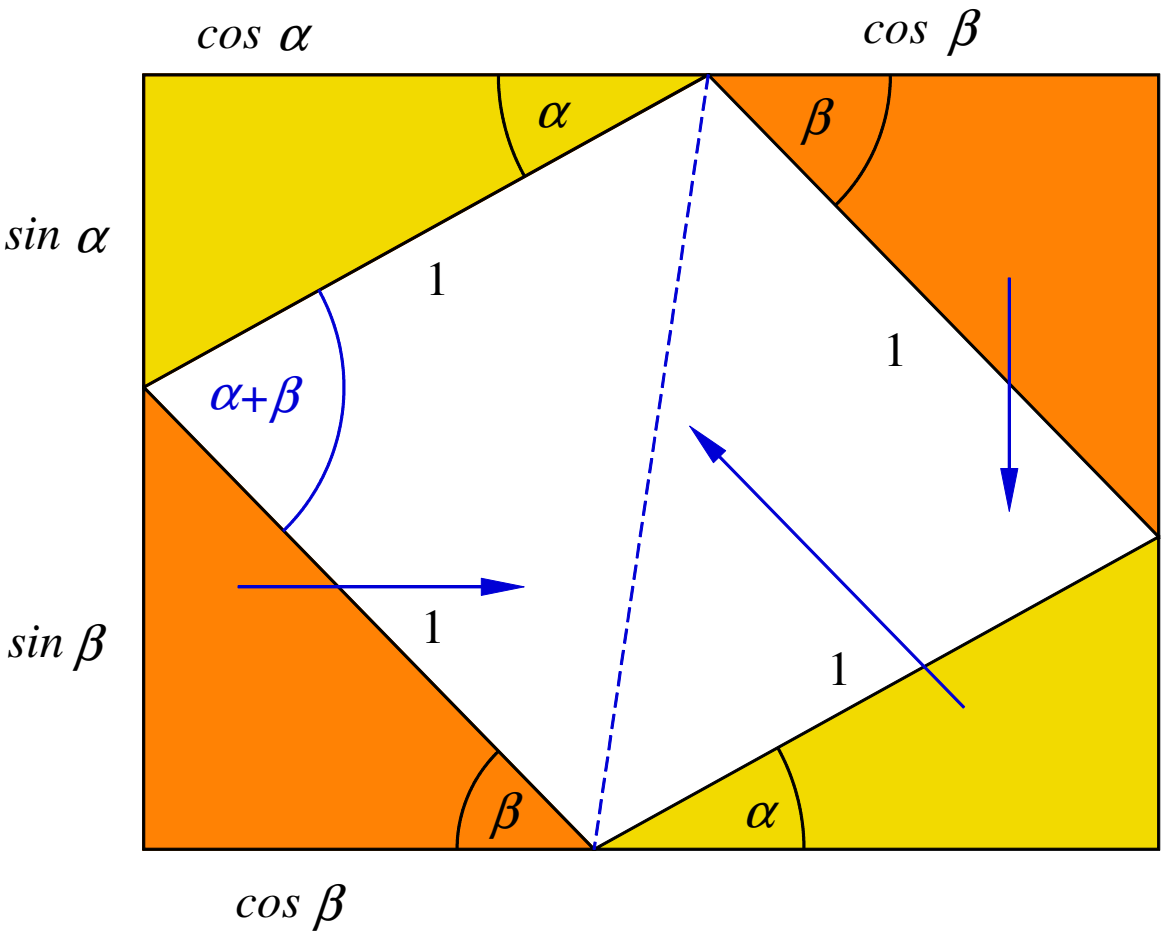


$$2 \cdot \frac{1}{2} \cdot 1 \cdot 1 \cdot \sin(\alpha+\beta) = \sin(\alpha+\beta)$$

$$\sin(\alpha+\beta) = \sin\alpha \cdot \cos\beta + \cos\alpha \cdot \sin\beta$$



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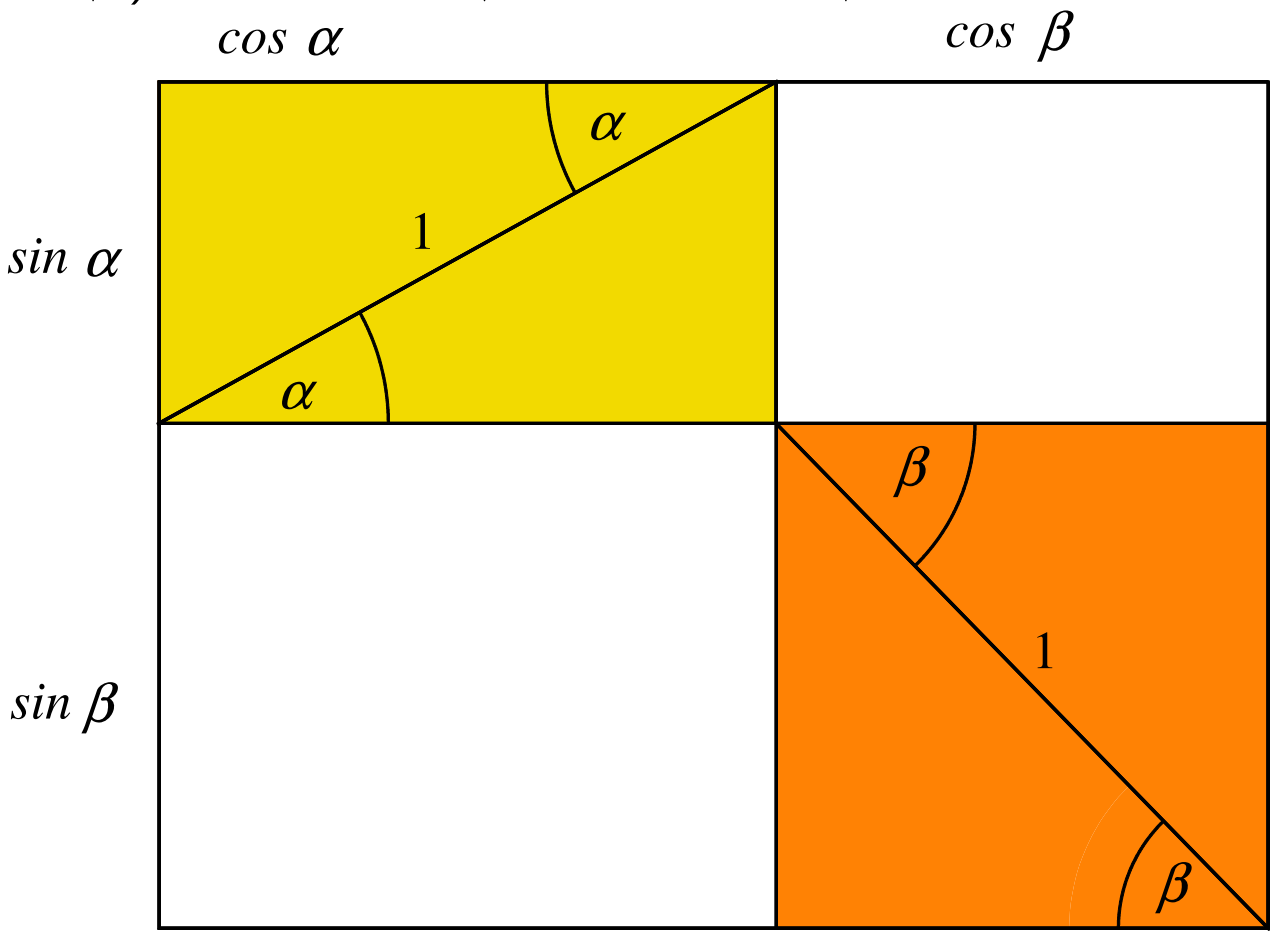
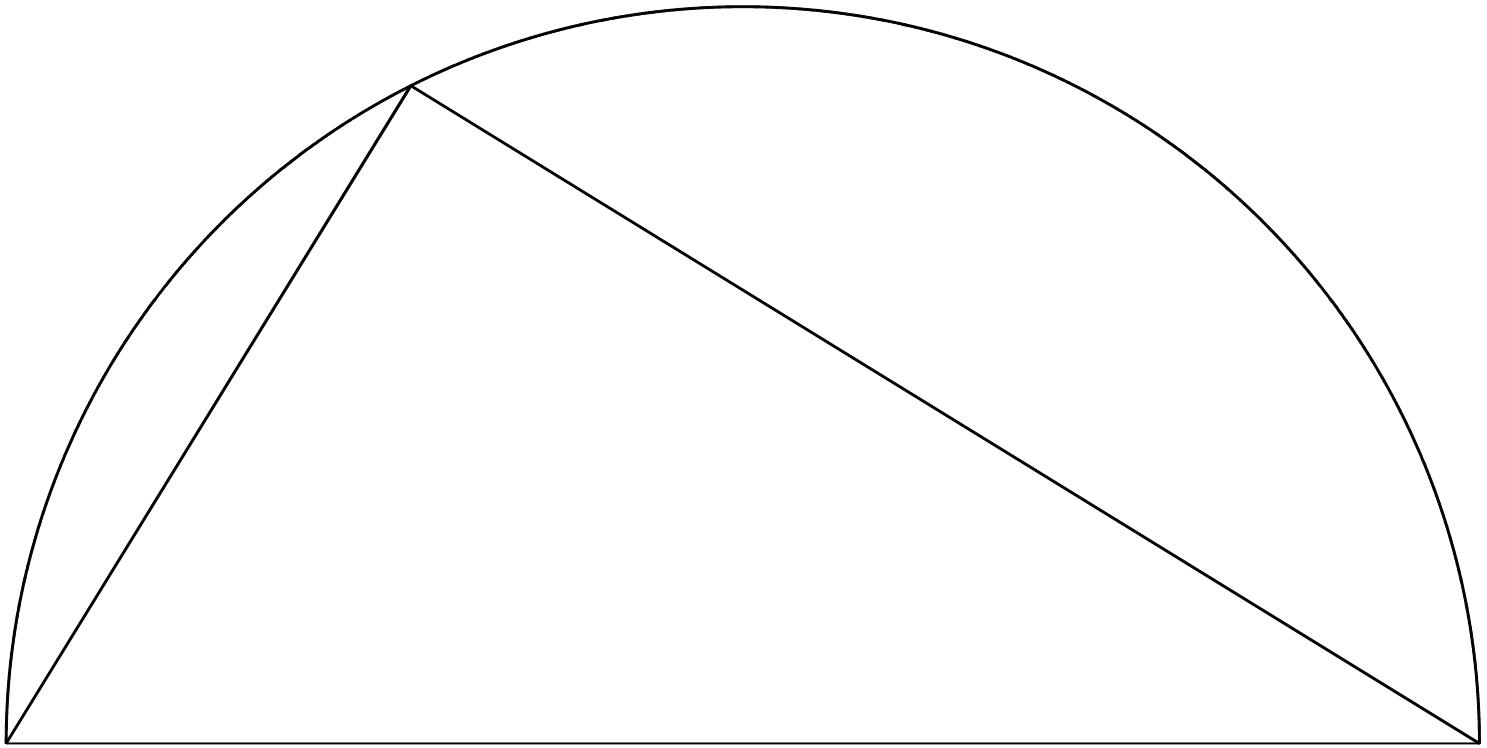


AG nerovnost

Nechť x, y jsou kladná reálná čísla. Potom $\frac{x+y}{2} \geq \sqrt{xy}$.



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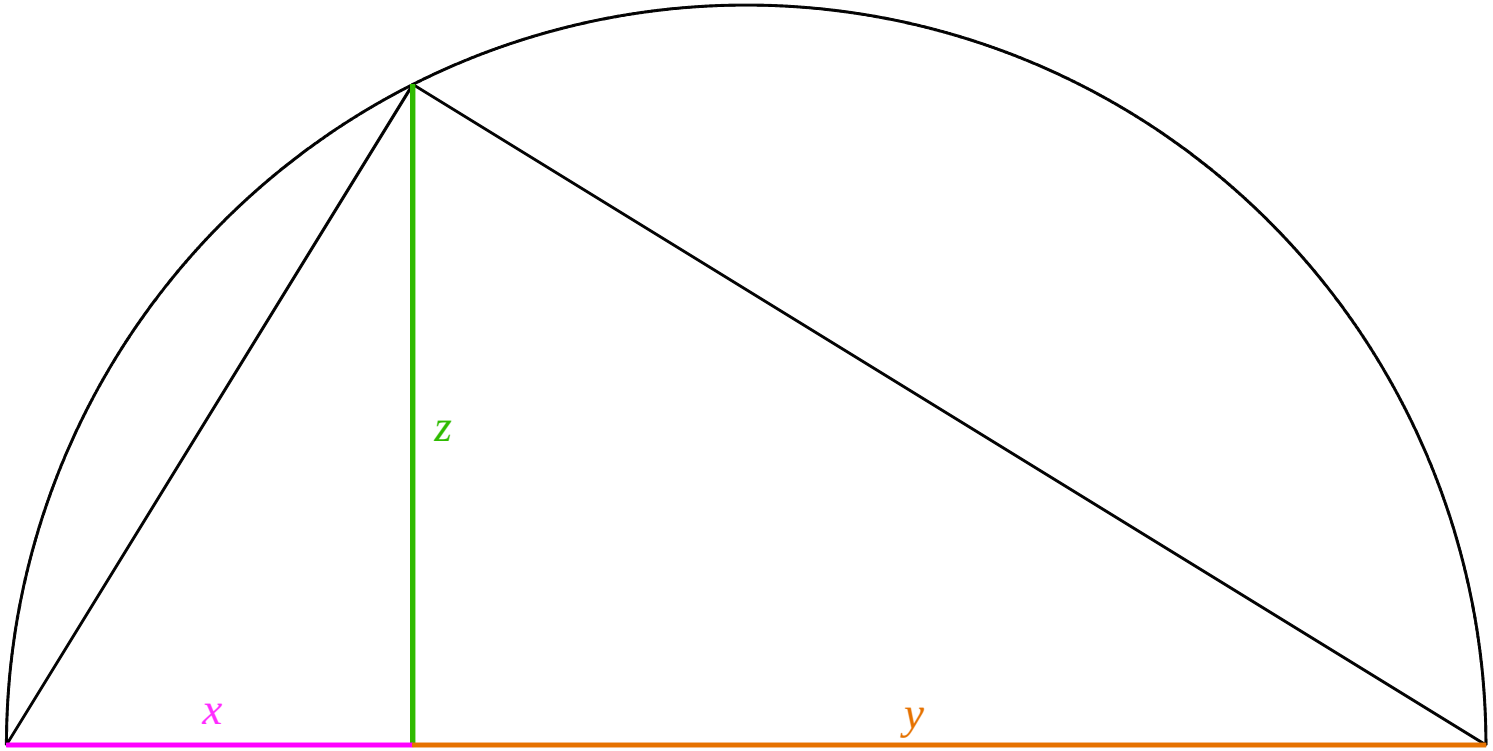

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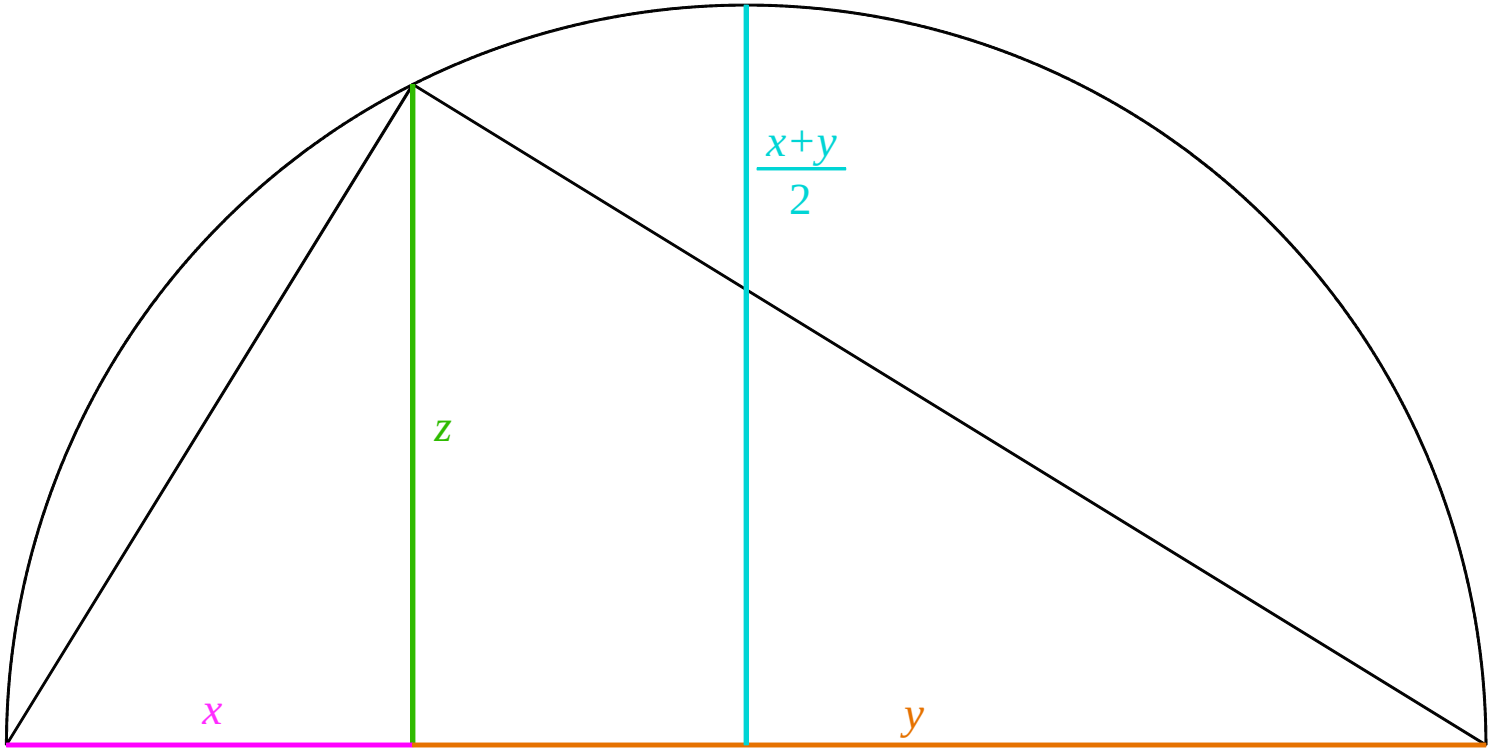

AG nerovnost – 2. způsob

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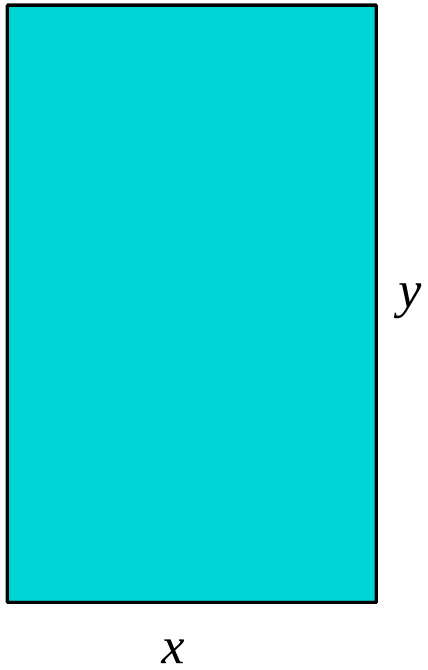

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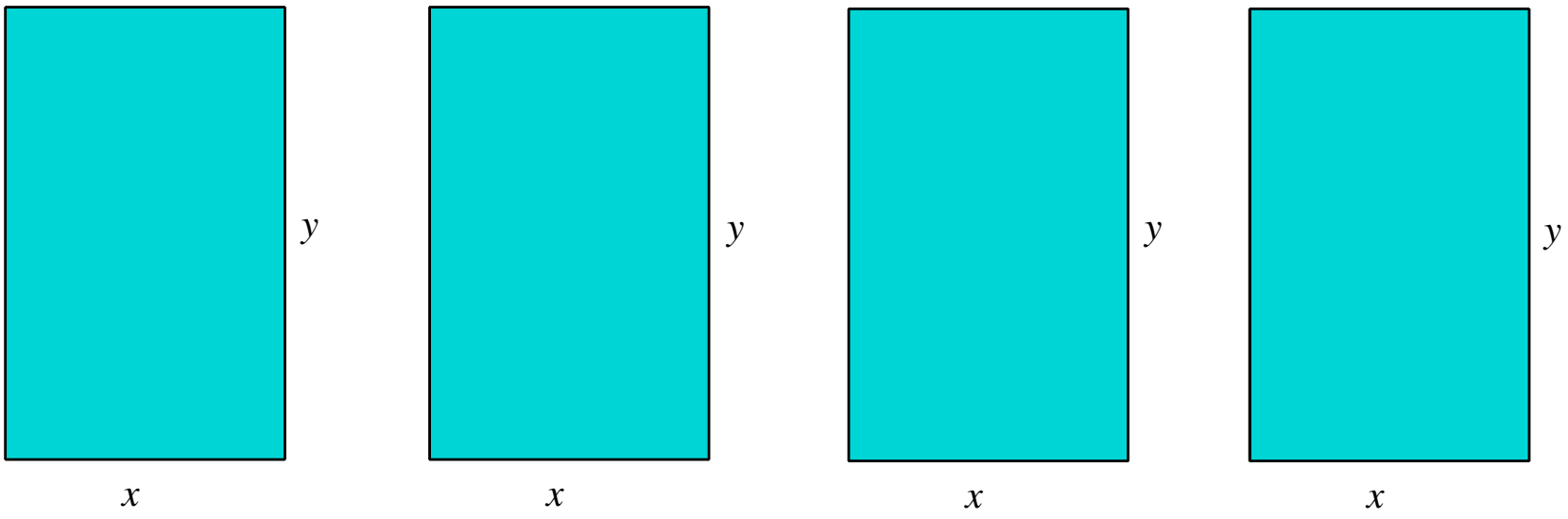

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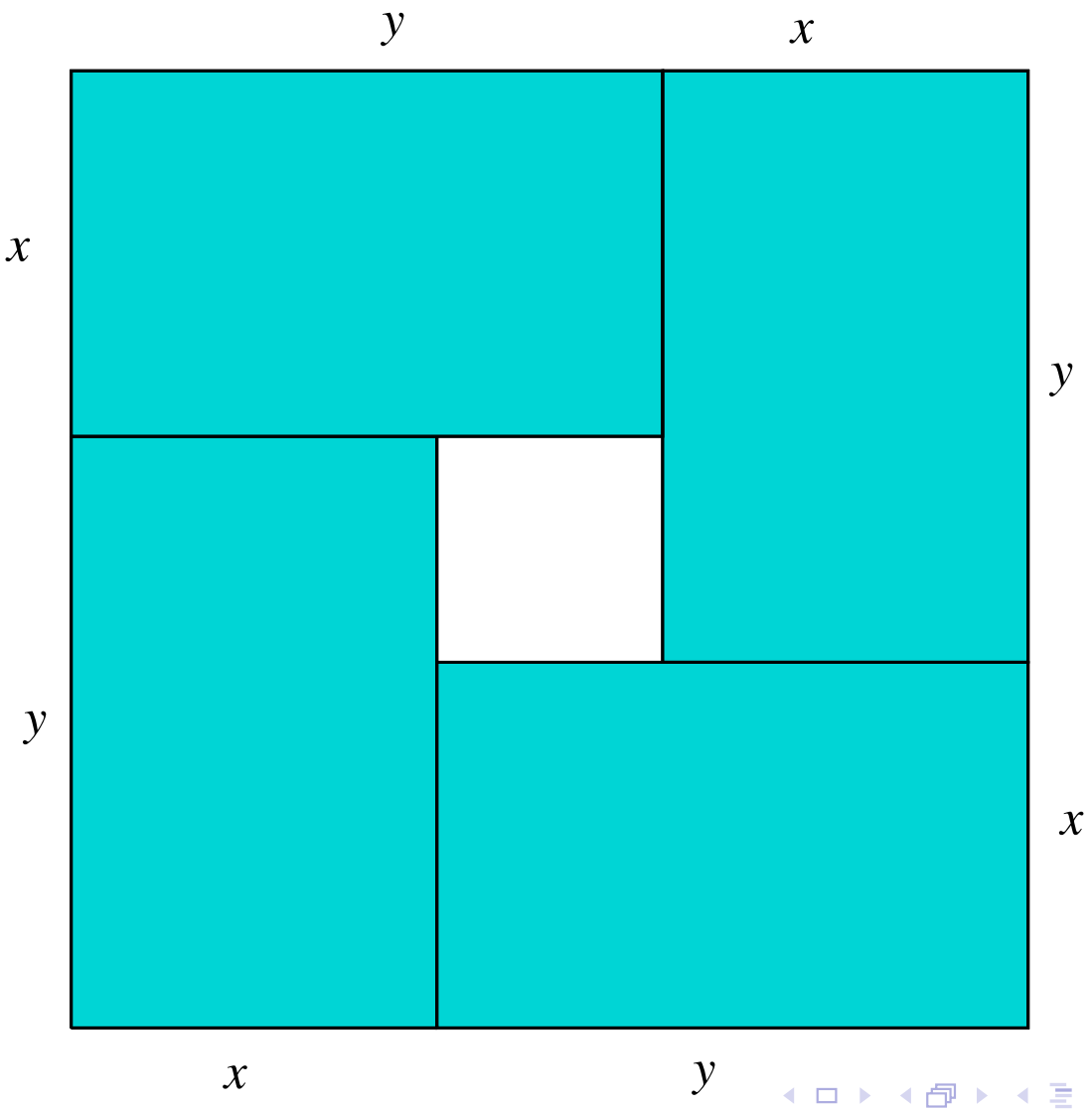
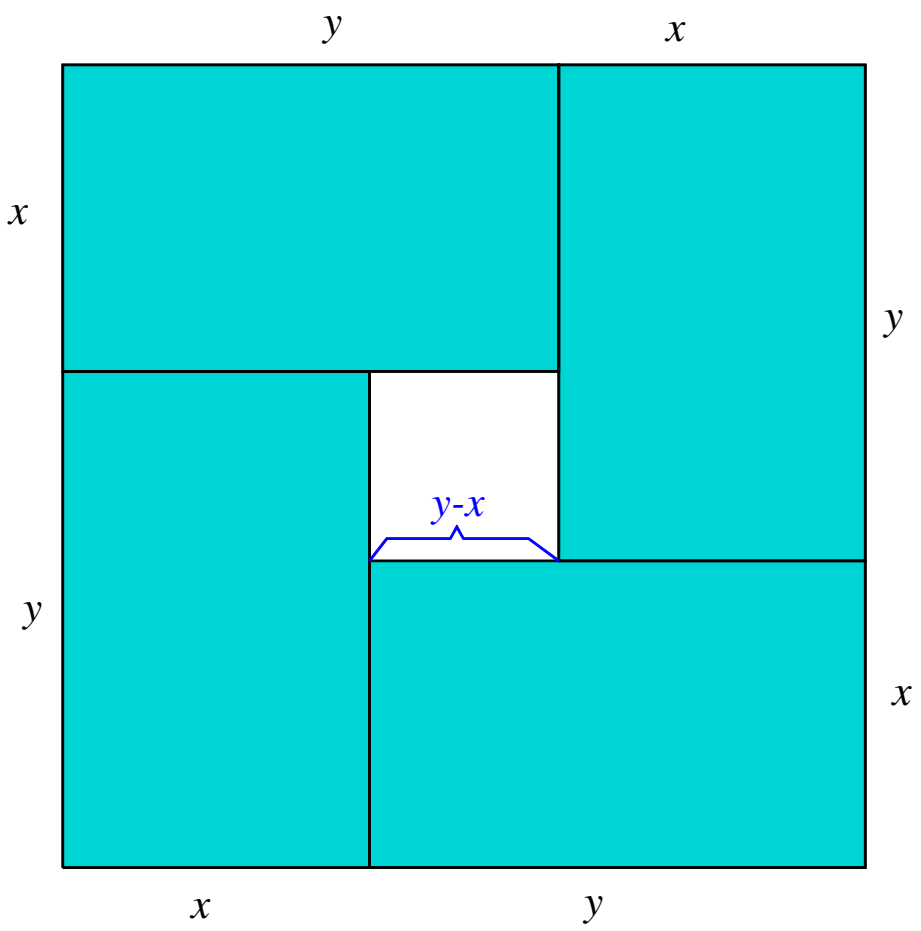
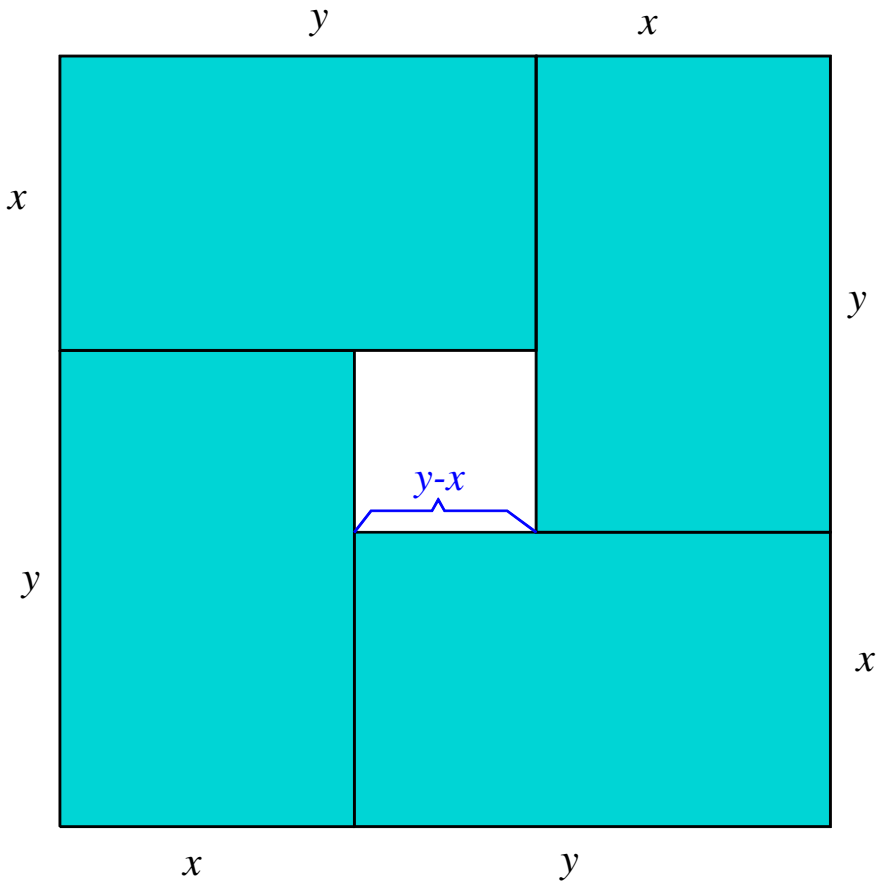




$$4xy = (x+y)^2 - (y-x)^2 \quad \Longrightarrow \quad 4xy \leq (x+y)^2$$



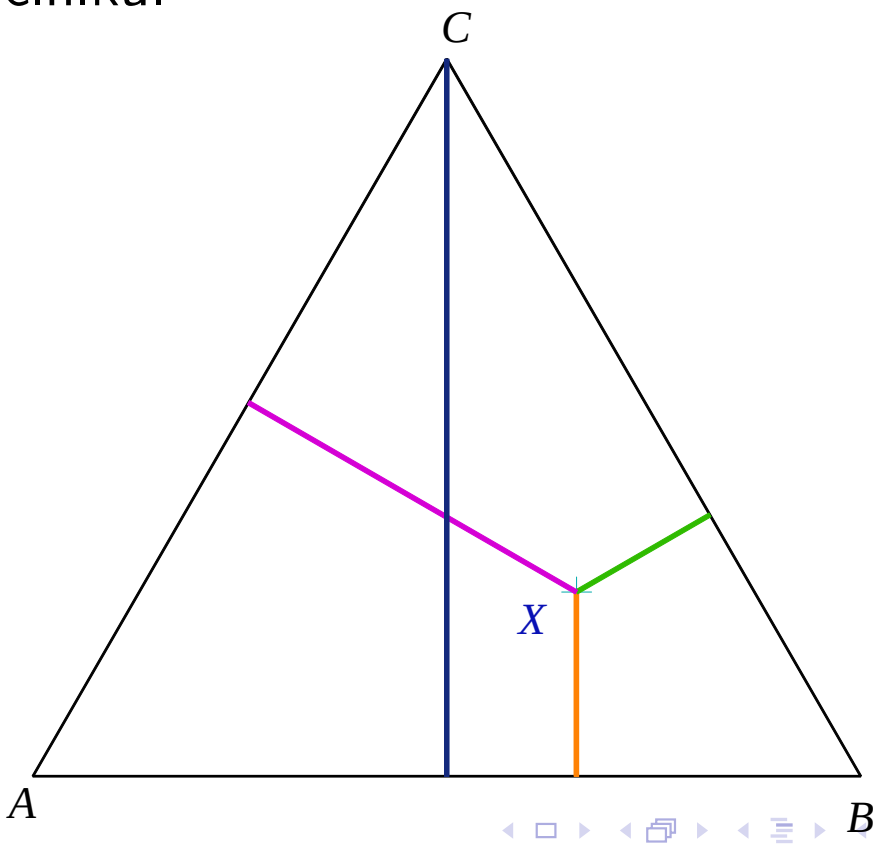
$$4xy \leq (x+y)^2,$$

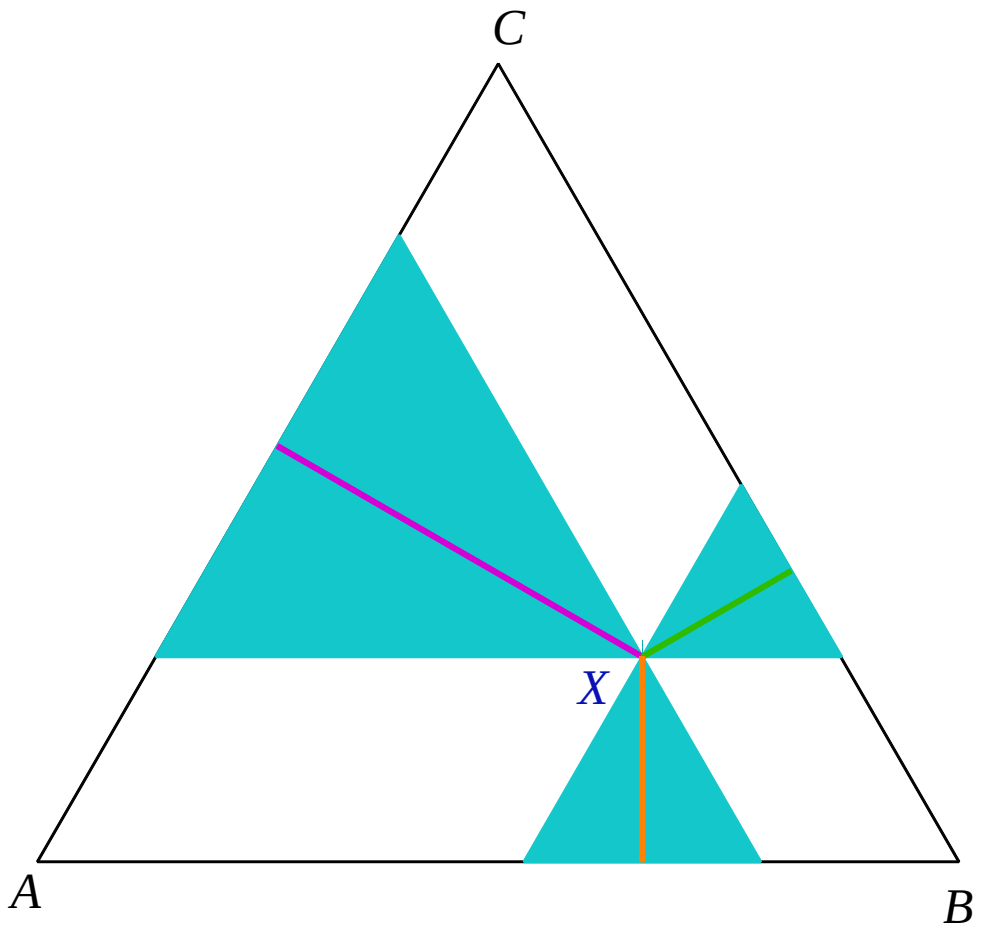
$$\sqrt{4xy} \leq x+y,$$

$$\sqrt{xy} \leq \frac{x+y}{2}.$$

Vivianiho věta

Nechť X je libovolný vnitřní bod rovnostranného trojúhelníku. Potom součet jeho vzdáleností od jednotlivých stran trojúhelníku je roven výšce tohoto trojúhelníku.




C
X
A
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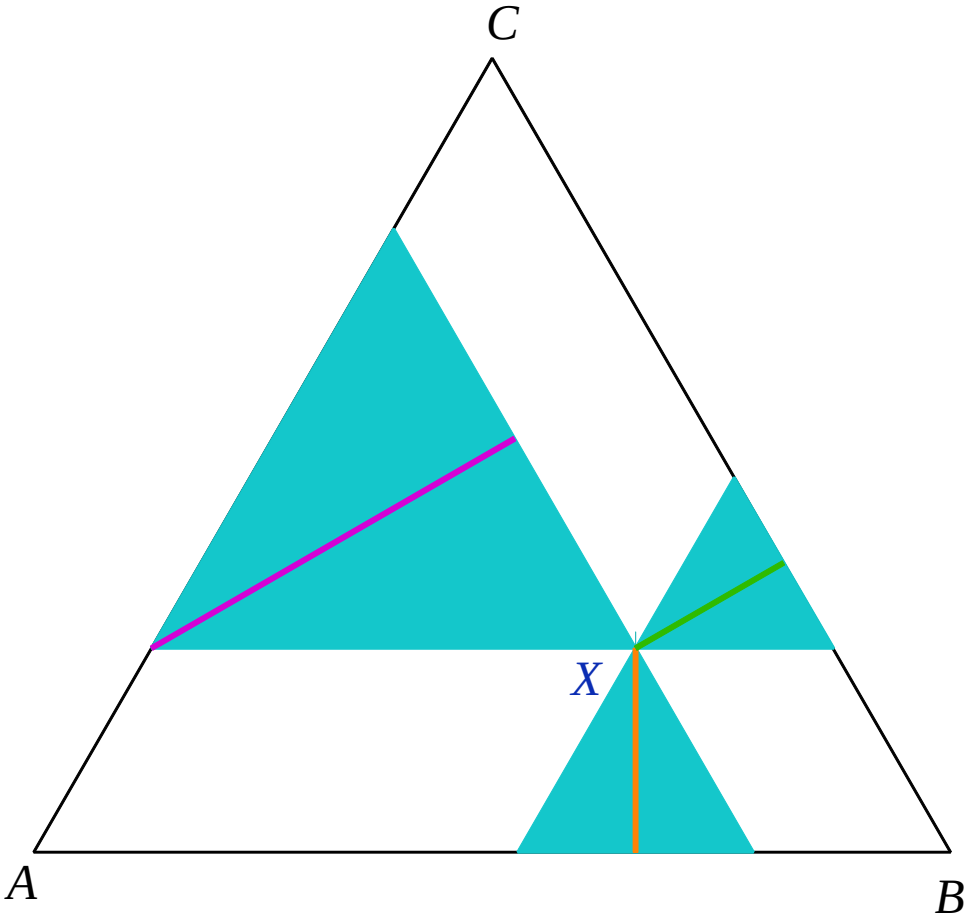
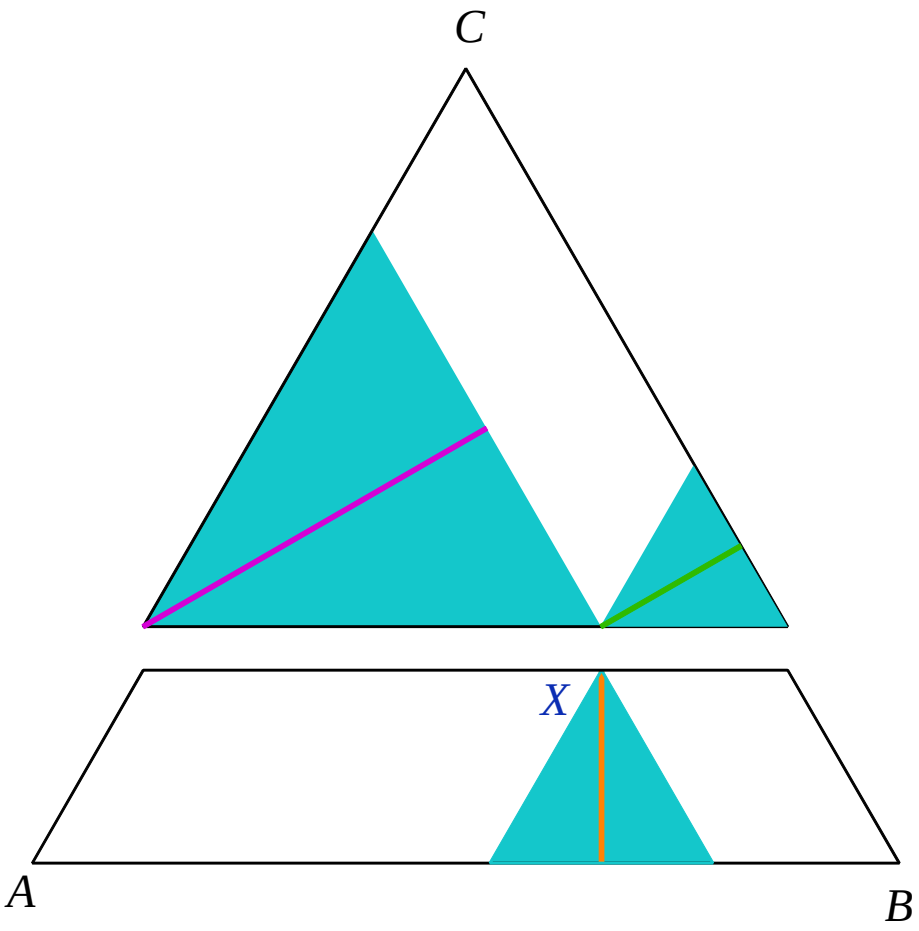

C
X
A
B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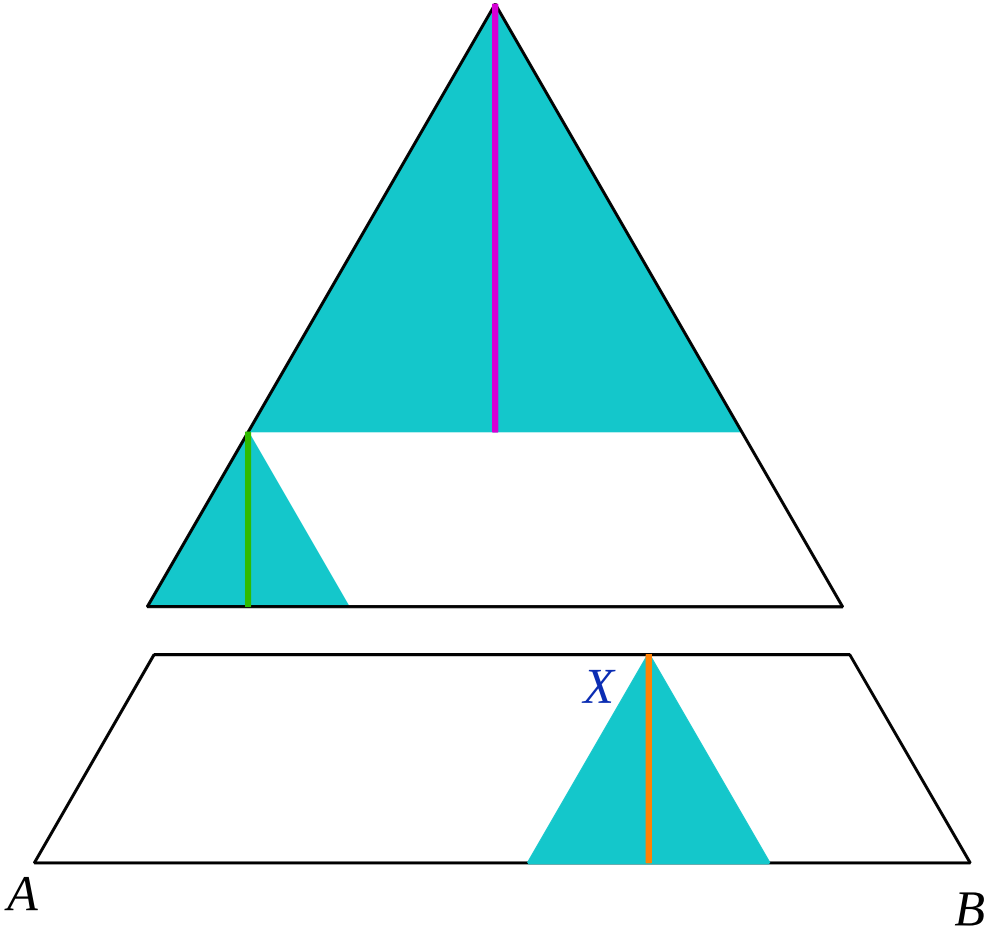
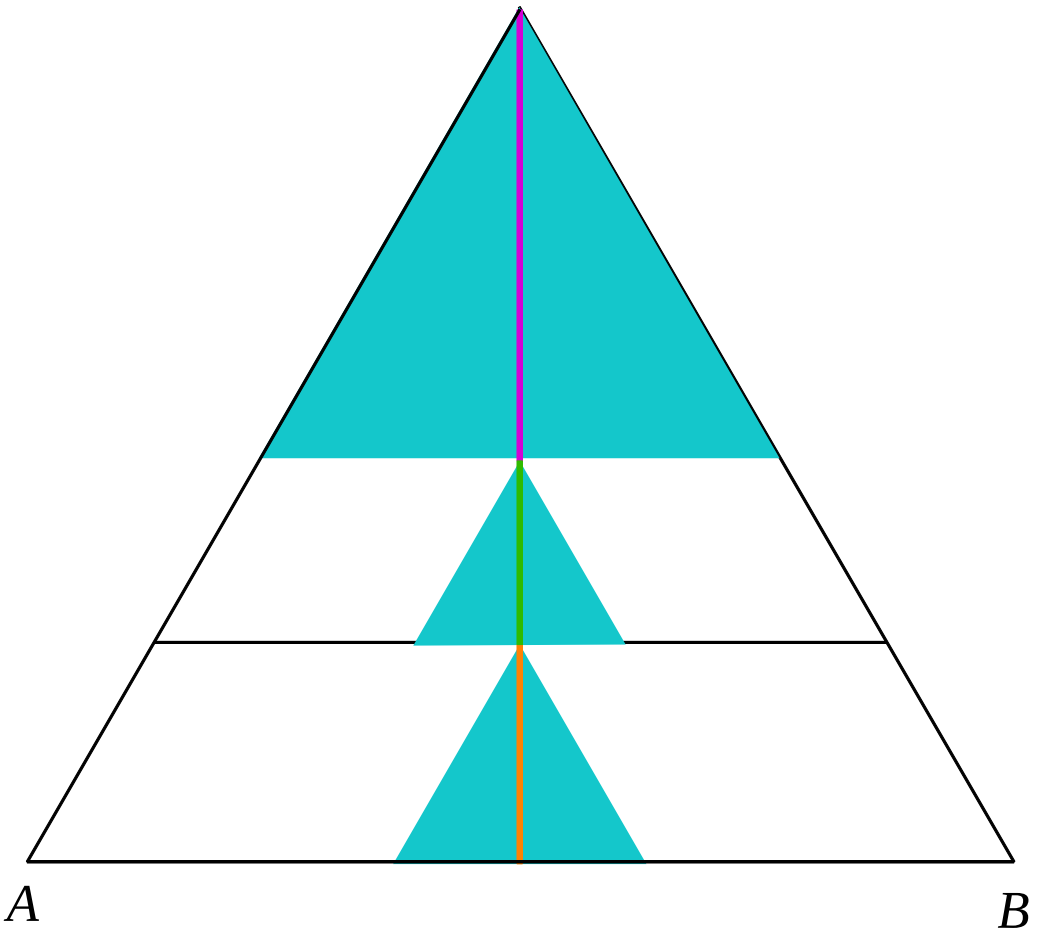
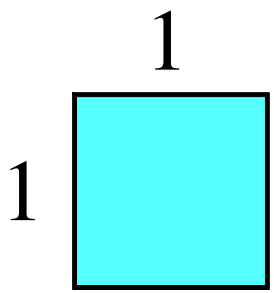

X
A
B


A
B

Součet přirozených čísel

Pro přirozené číslo n platí $1 + 2 + 3 + \cdots + n = \frac{n\cdot(n+1)}{2}$.



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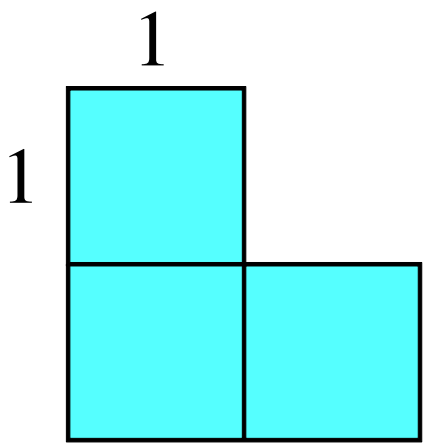

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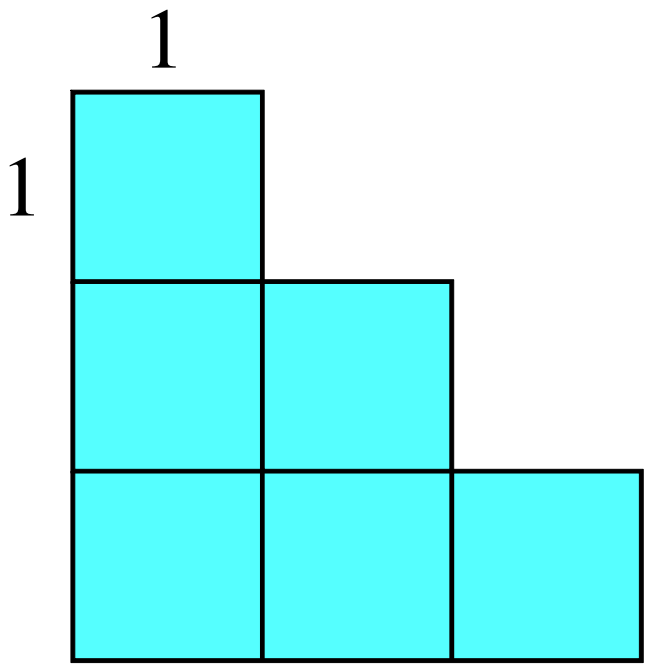

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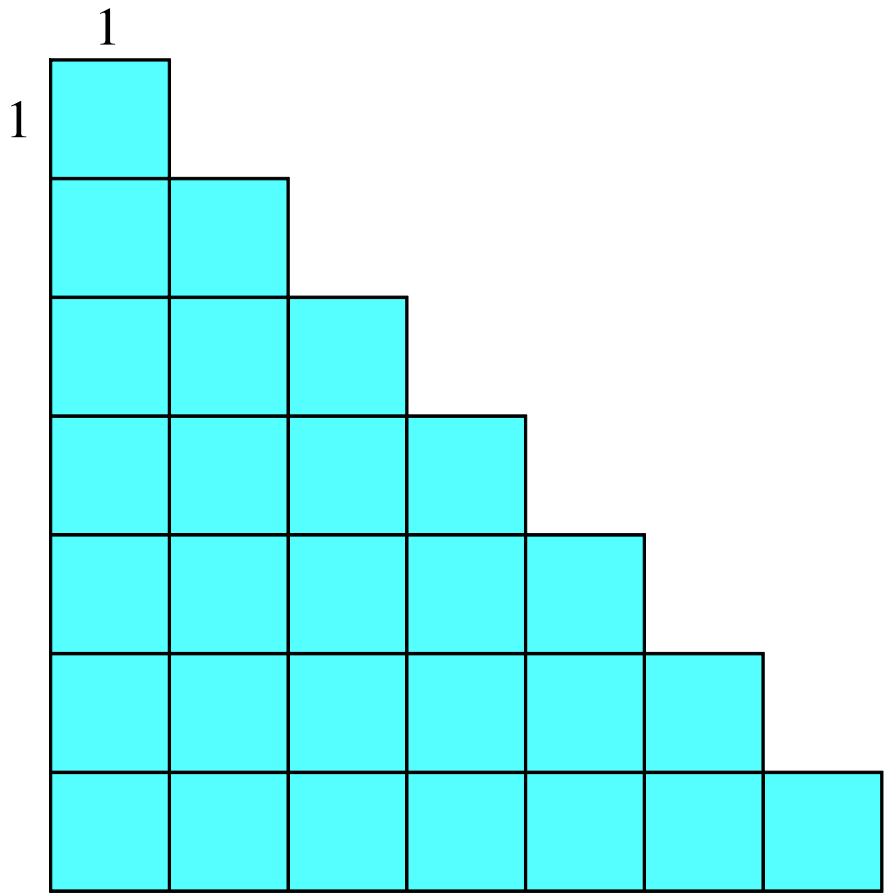

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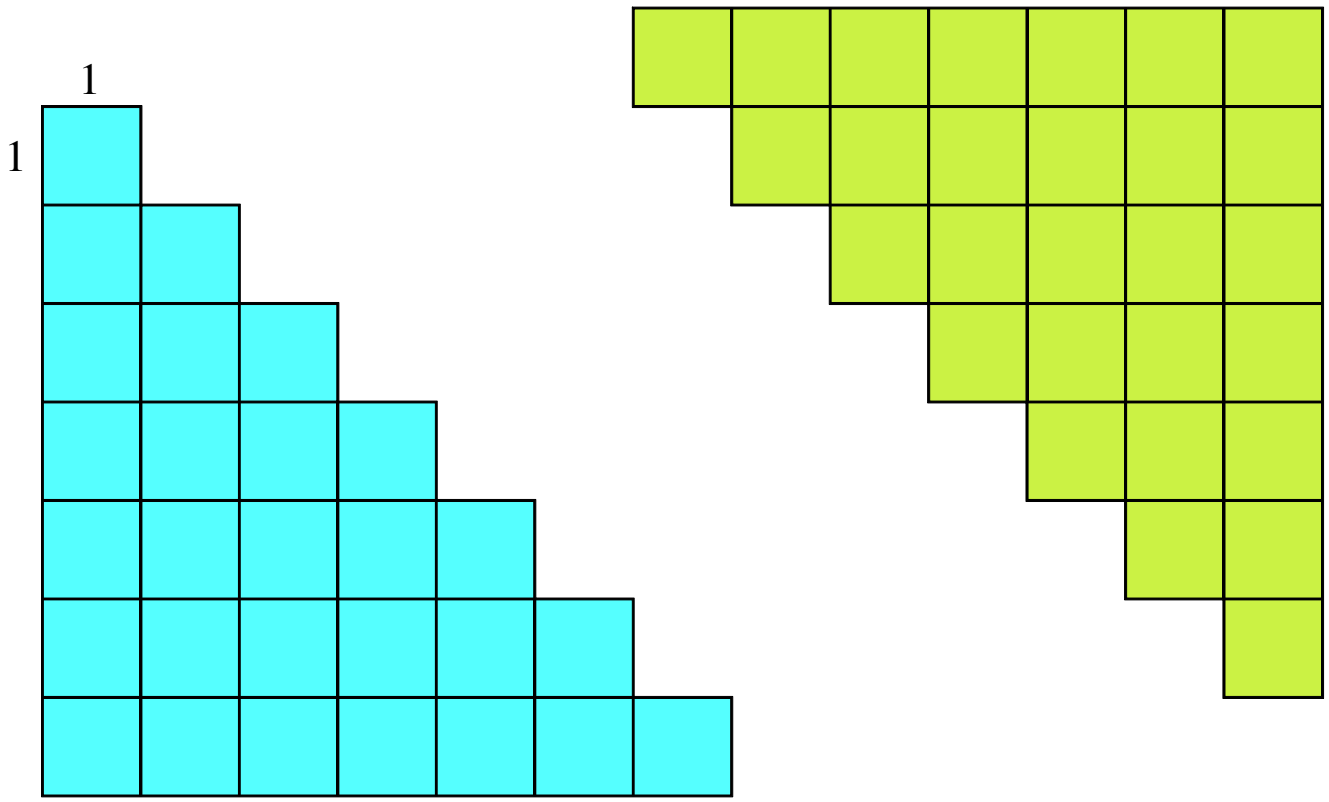

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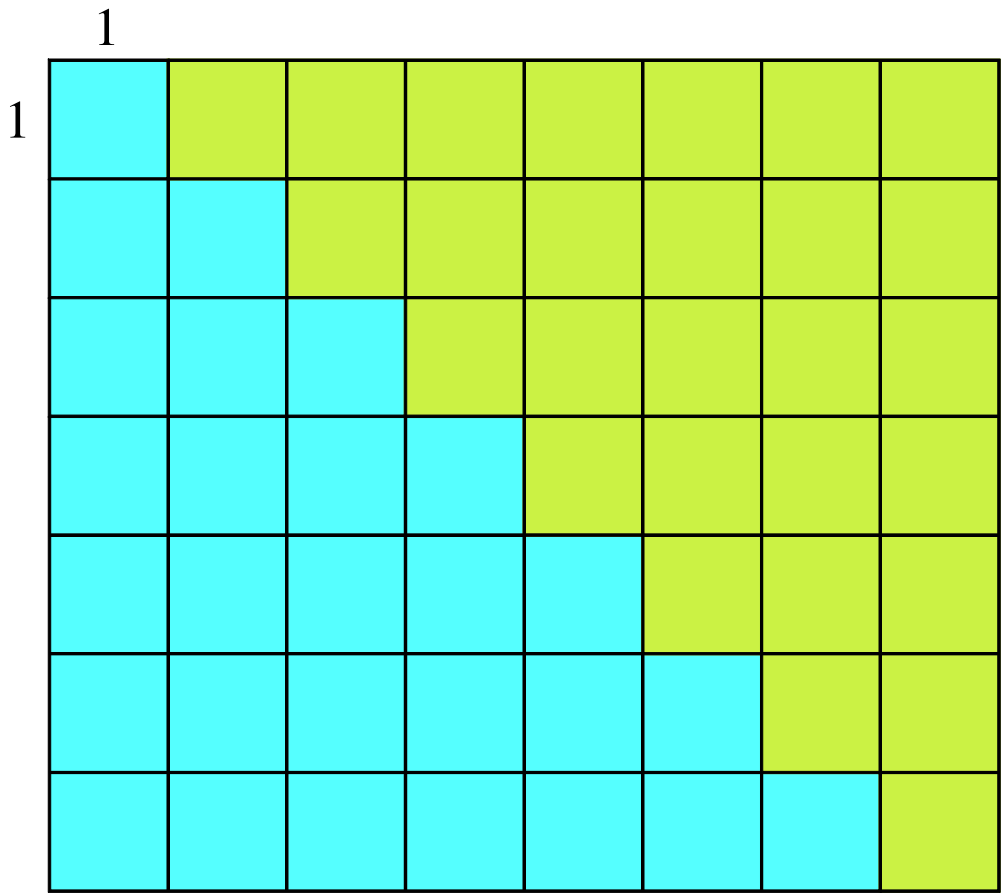

Součet přirozených čísel – 2. způsob

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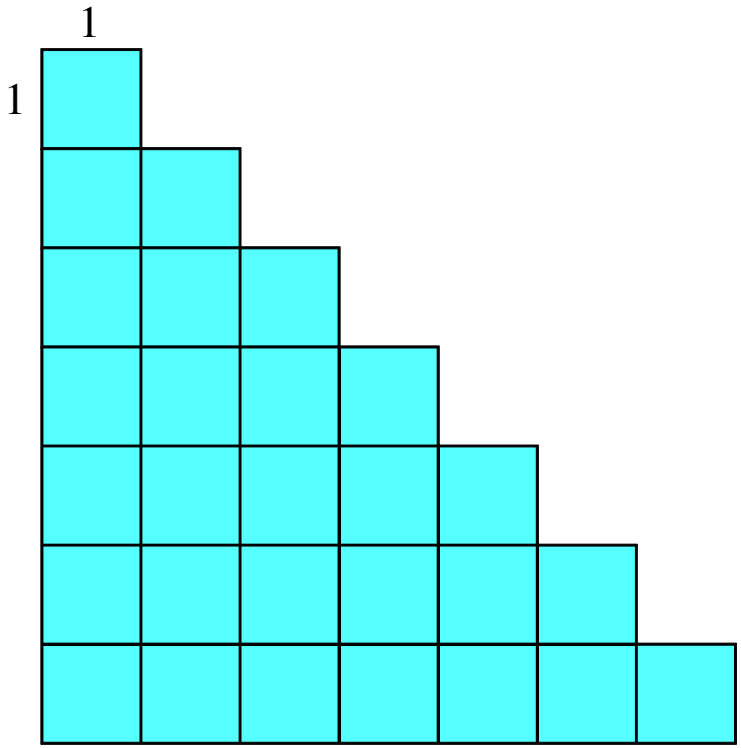
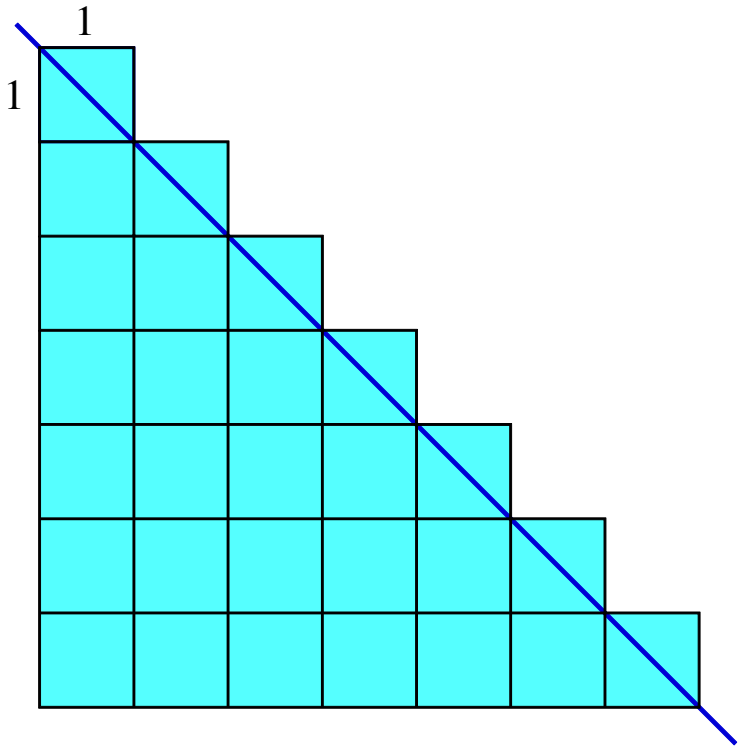
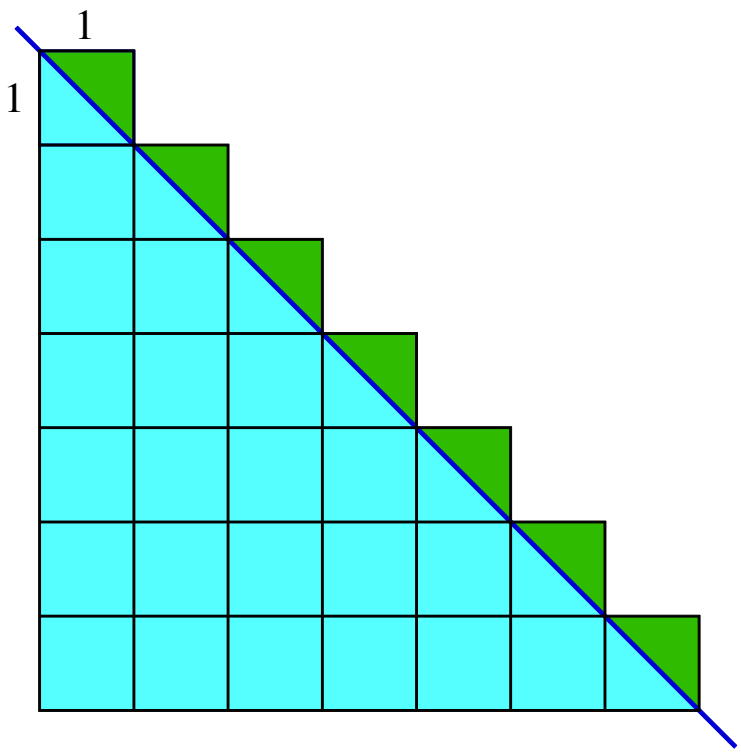
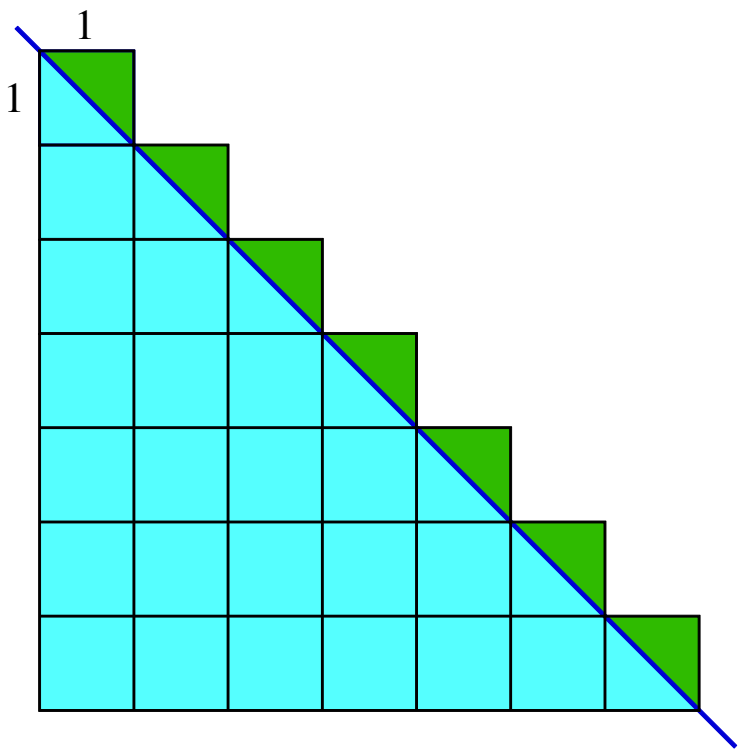
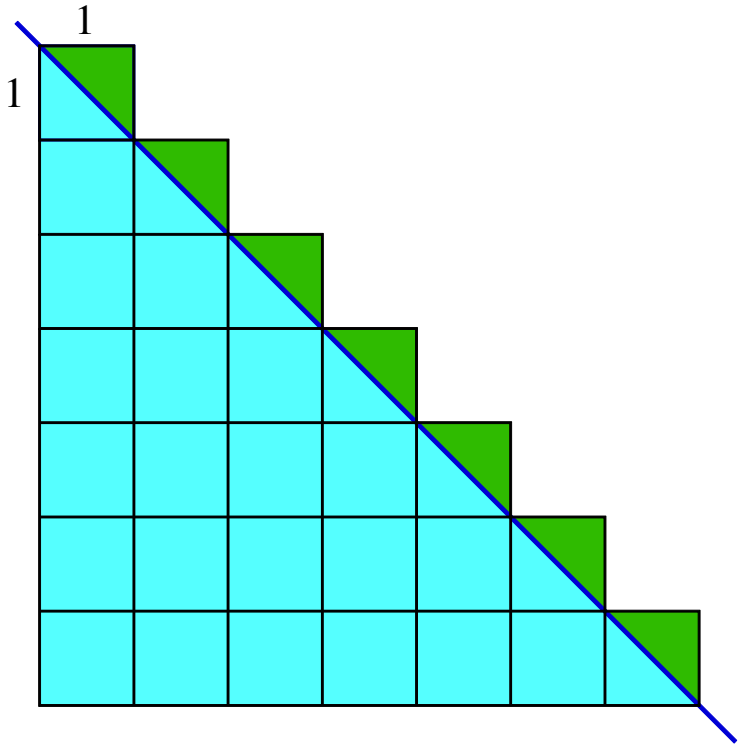



1
1


1
1



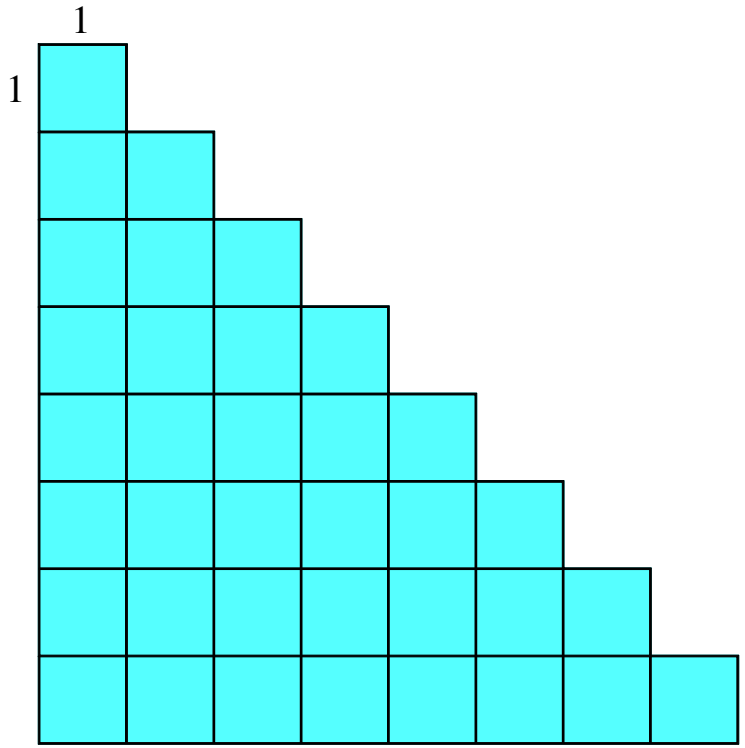
$$\frac{n^2}{2} + n \cdot \frac{1}{2} =$$



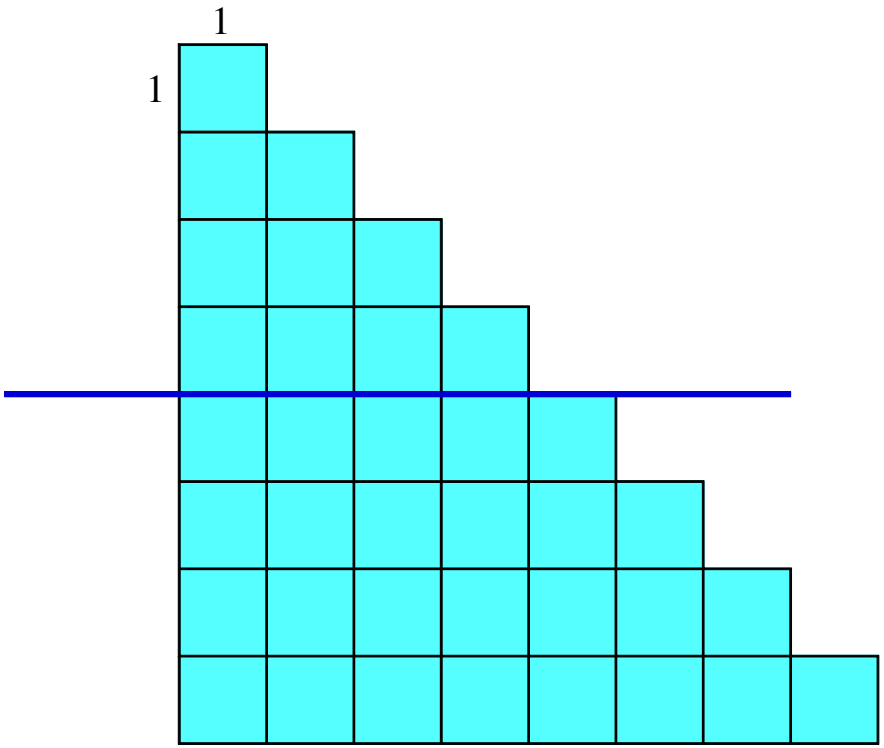
$$\frac{n^2}{2} + n \cdot \frac{1}{2} = \frac{n}{2} \cdot (n+1)$$

Součet přirozených čísel – 3. způsob pro n sudé

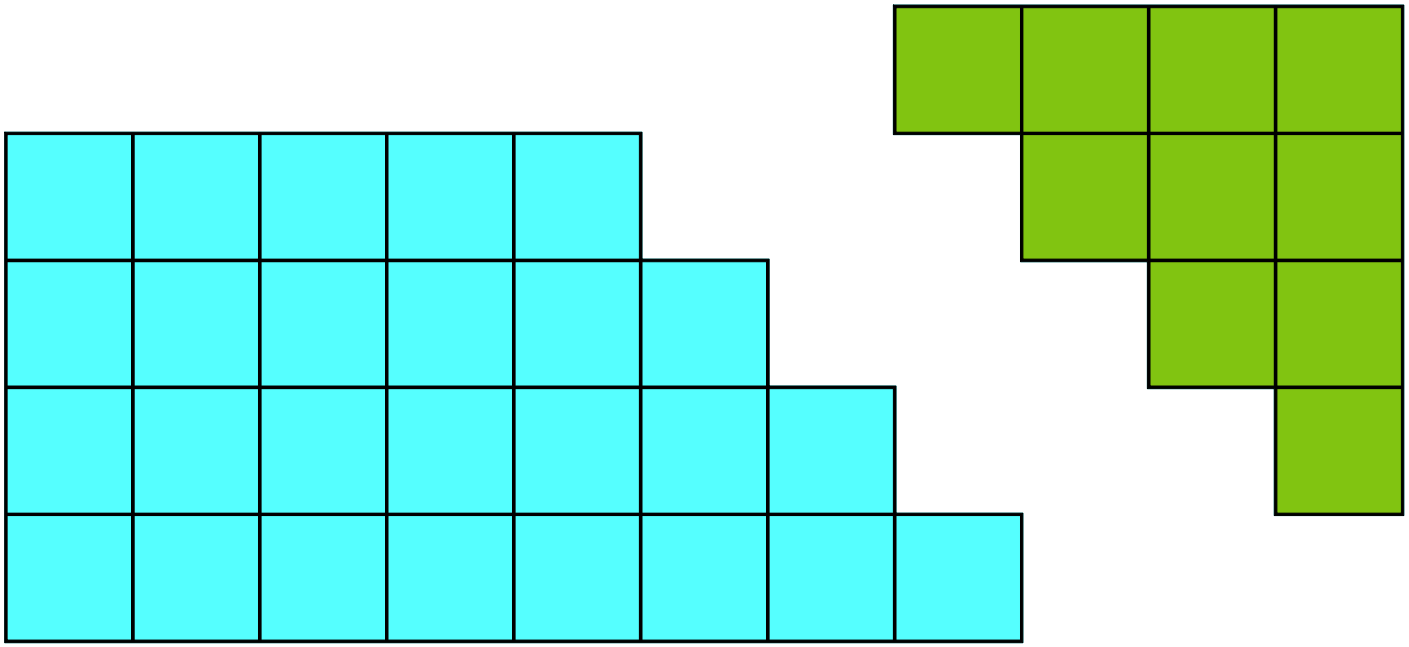
Pro přirozené číslo n platí $1 + 2 + 3 + \cdots + n = \frac{n\cdot(n+1)}{2}$.



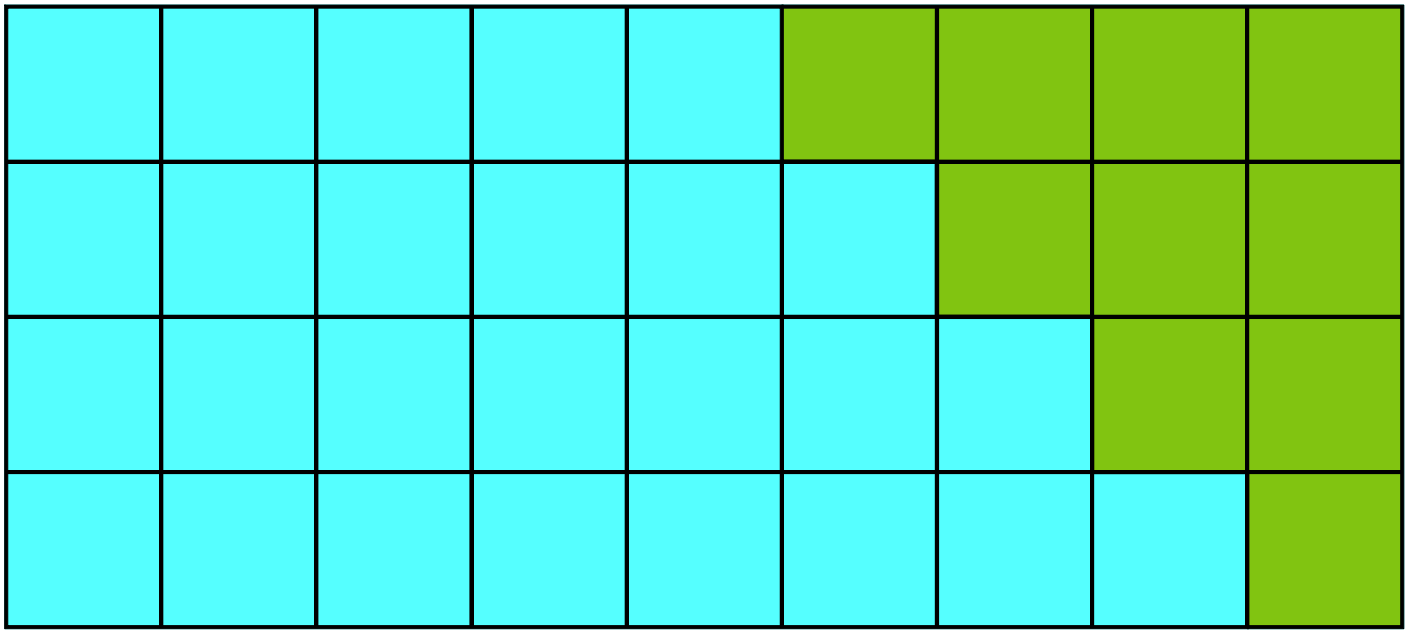
Pro přirozené číslo n platí $1 + 2 + 3 + \cdots + n = \frac{n\cdot(n+1)}{2}$.



Pro přirozené číslo n platí $1 + 2 + 3 + \cdots + n = \frac{n\cdot(n+1)}{2}$.

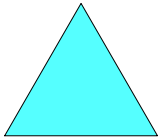


Pro přirozené číslo n platí $1 + 2 + 3 + \cdots + n = \frac{n\cdot(n+1)}{2}$.



Součet mocnin čísla 3

Pro přirozené číslo n platí $1 + 3 + 3^2 + \cdots + 3^n = \frac{3^{n+1}-1}{2}$.



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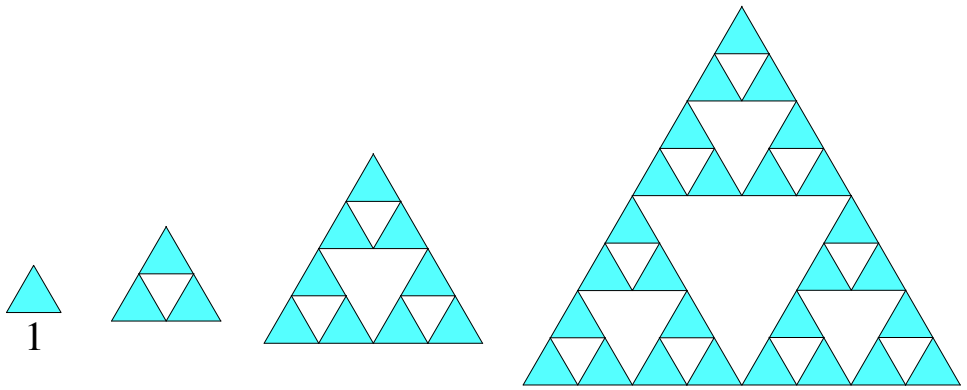
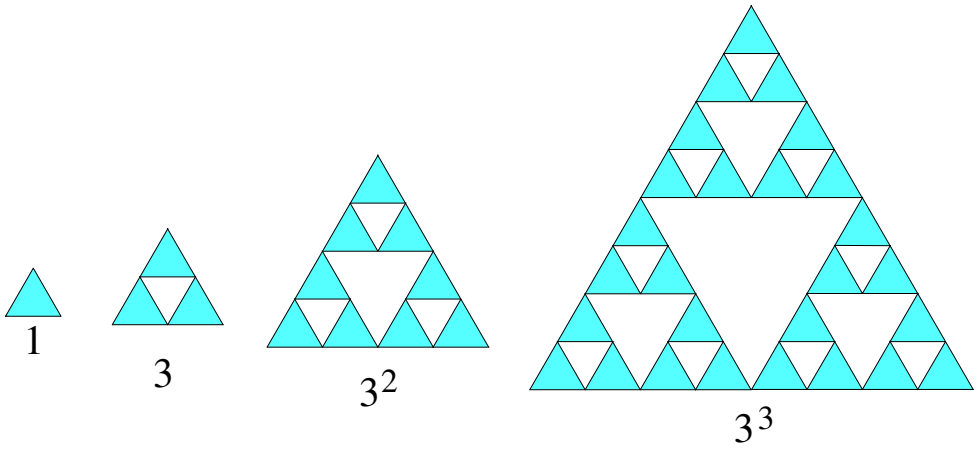
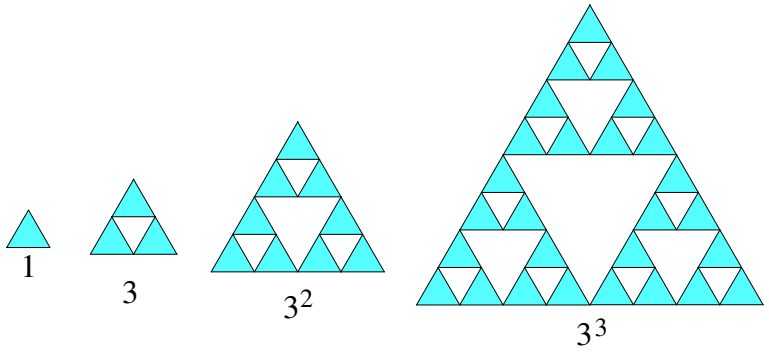
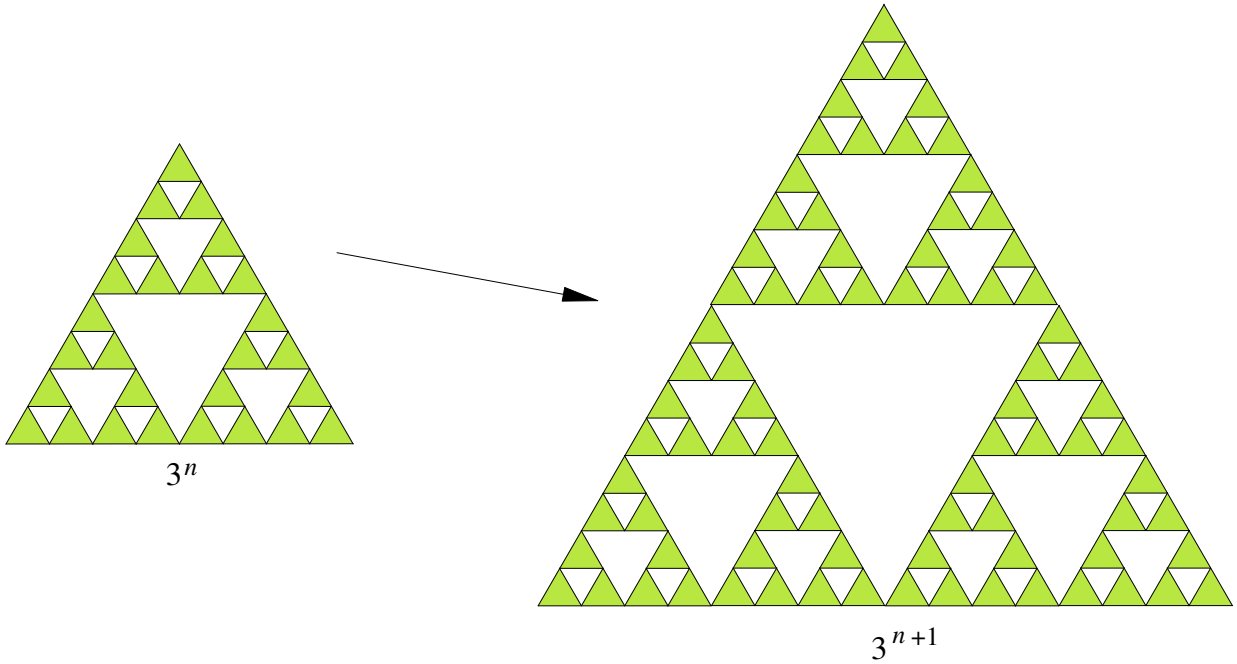
Pro přirozené číslo n platí $1 + 3 + 3^2 + \cdots + 3^n = \frac{3^{n+1}-1}{2}$.

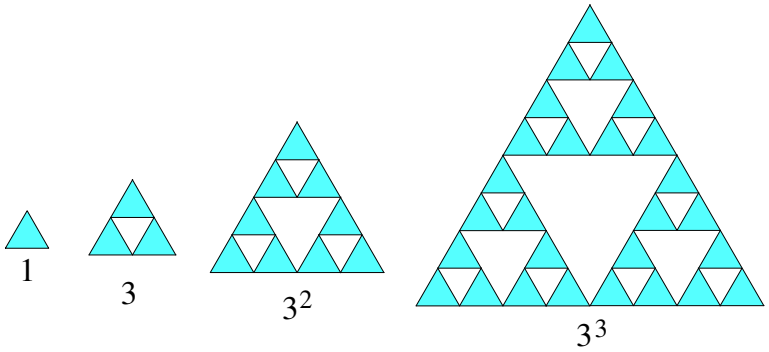


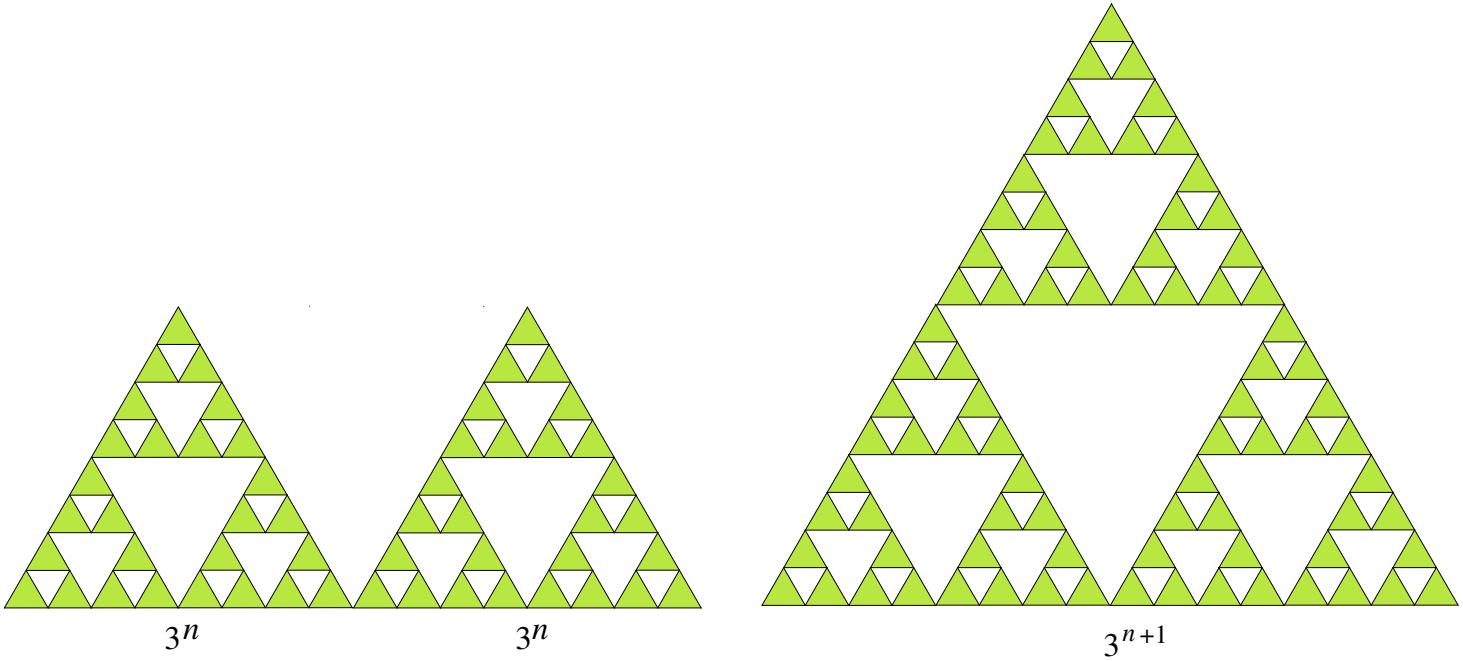
Pro přirozené číslo n platí $1 + 3 + 3^2 + \cdots + 3^n = \frac{3^{n+1}-1}{2}$.

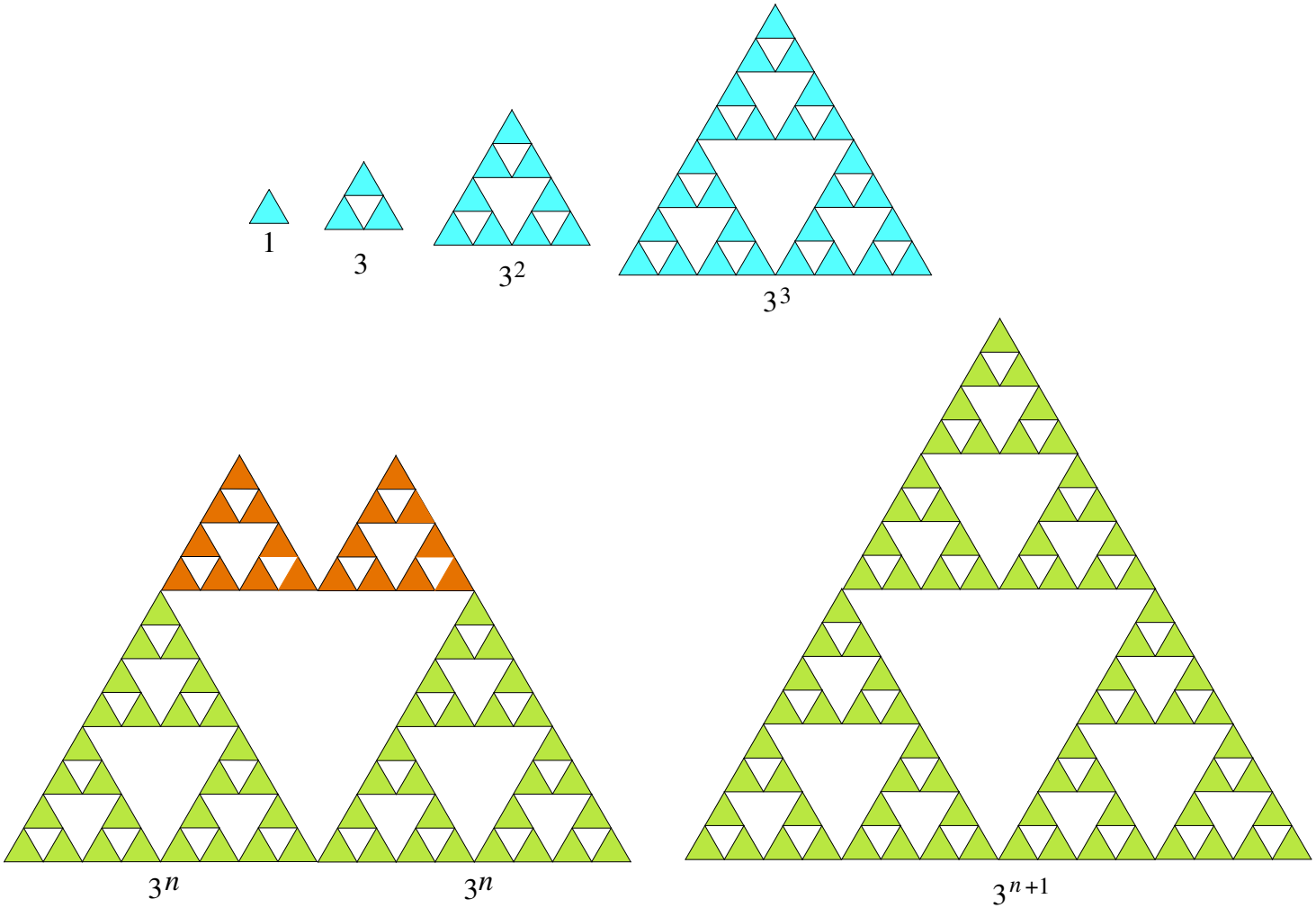


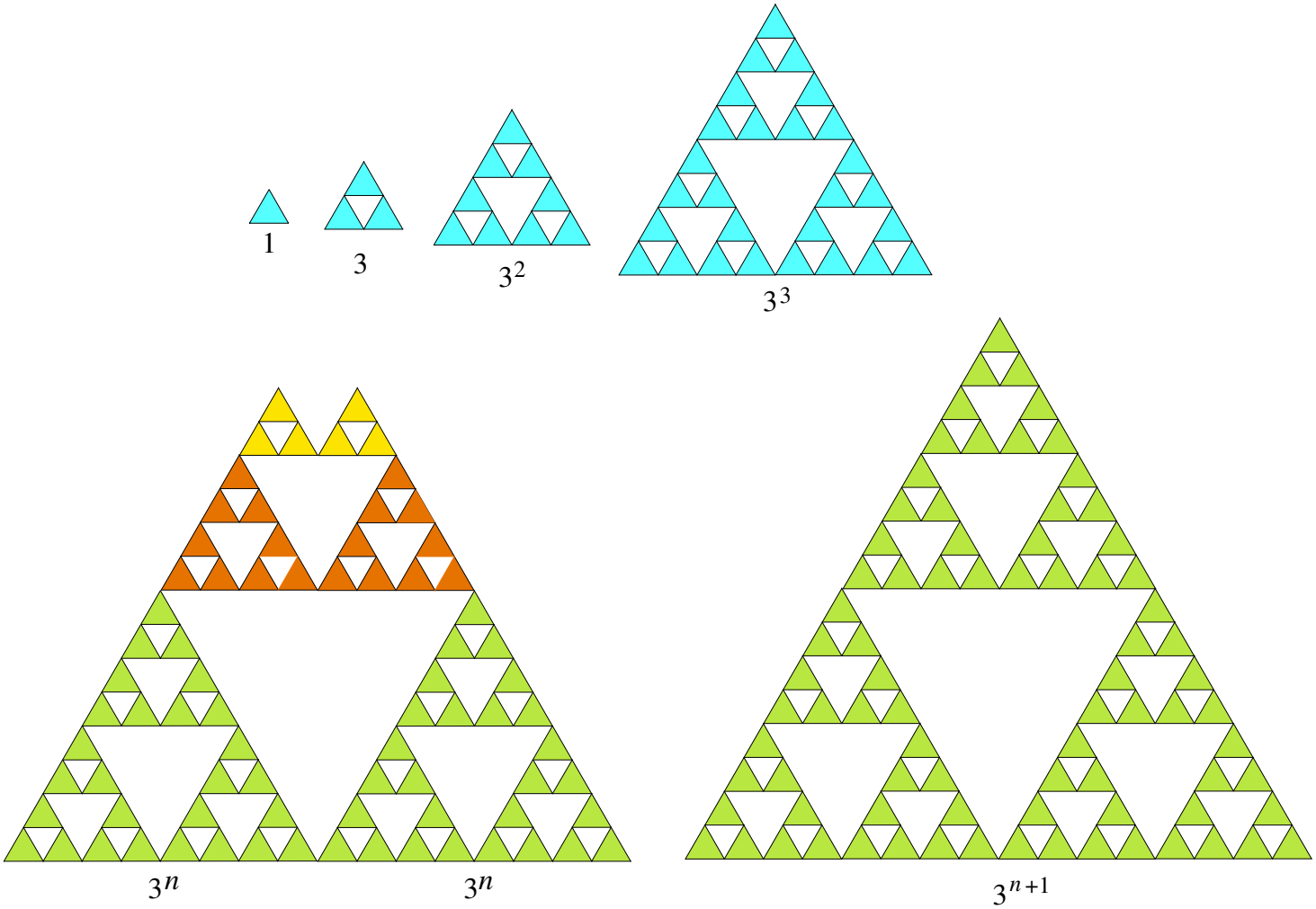

1
3
3^2
3^3

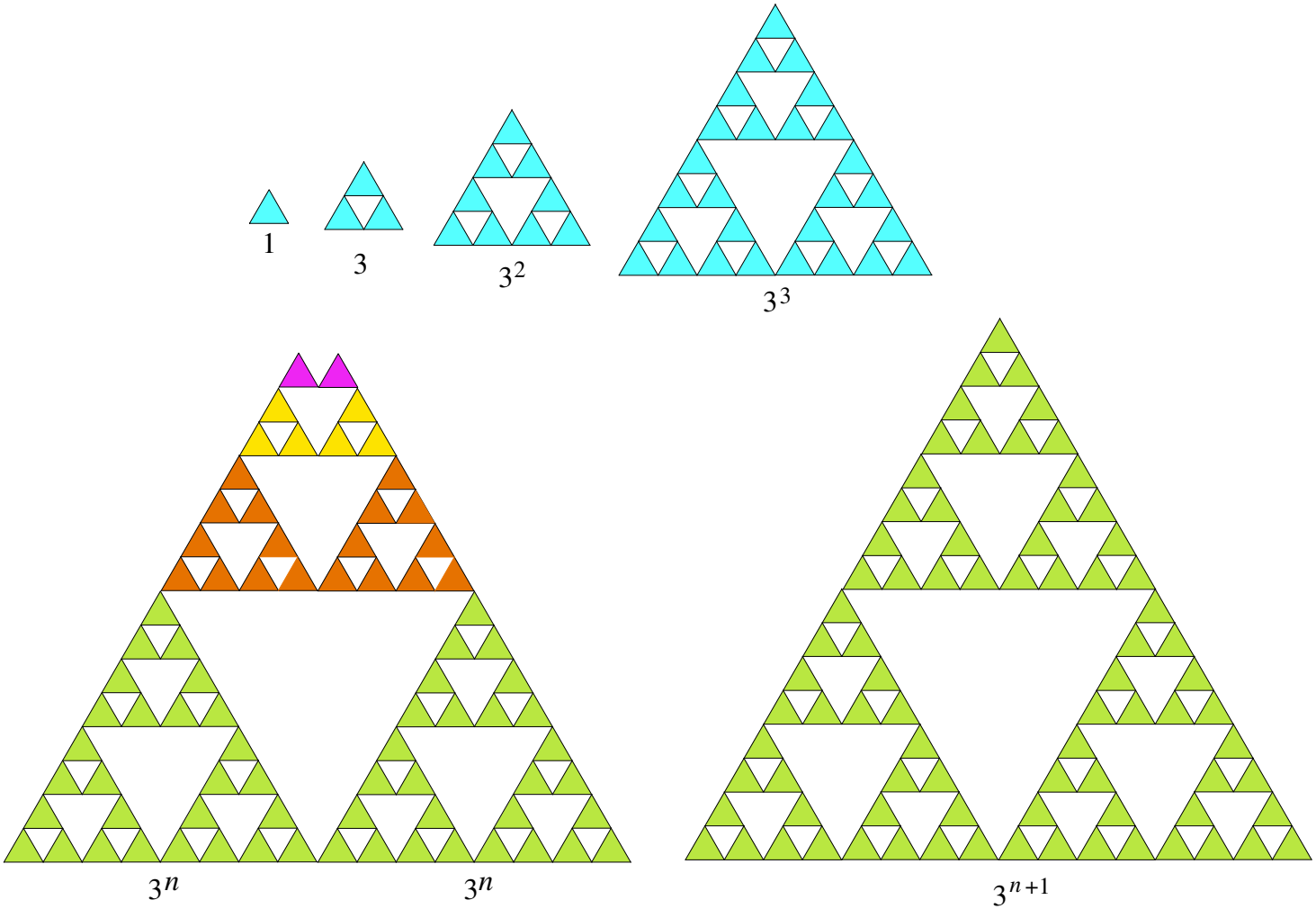

3^n
3^{n+1}

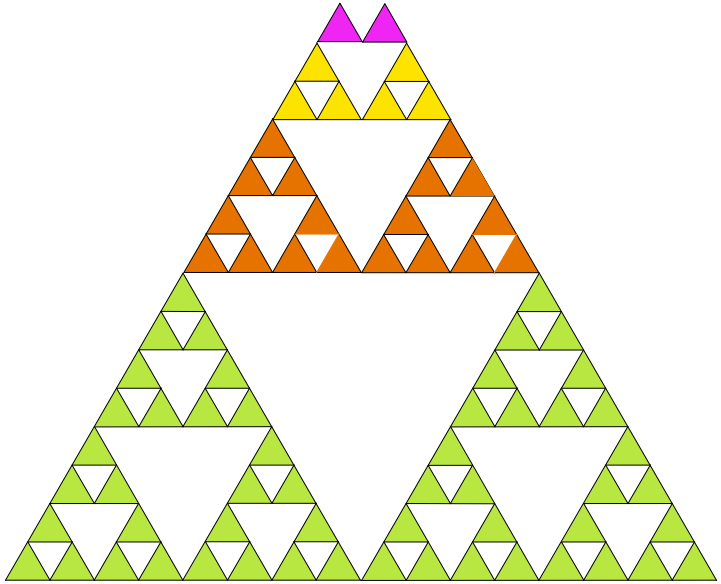

1
3
3^2
3^3


3^n
3^n
3^{n+1}

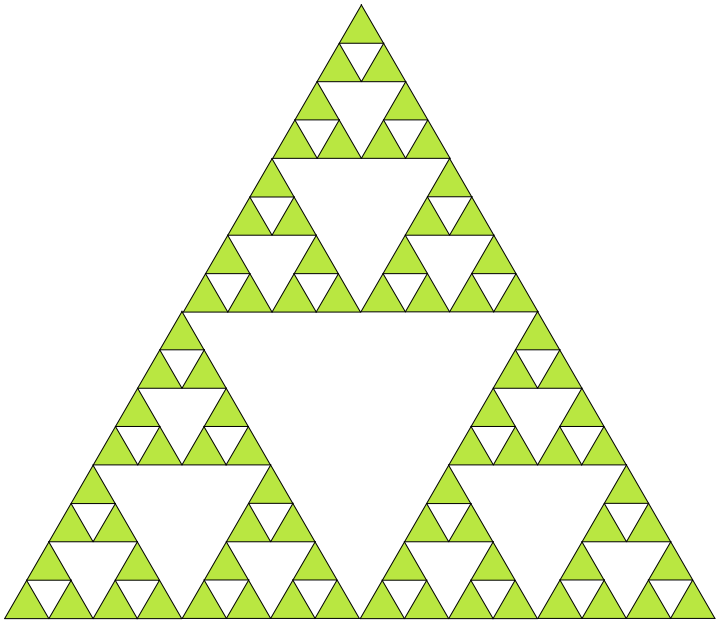

1
3
3^2
3^3
3^n
3^n
3^{n+1}


1
3
3^2
3^3
3^n
3^n
3^{n+1}

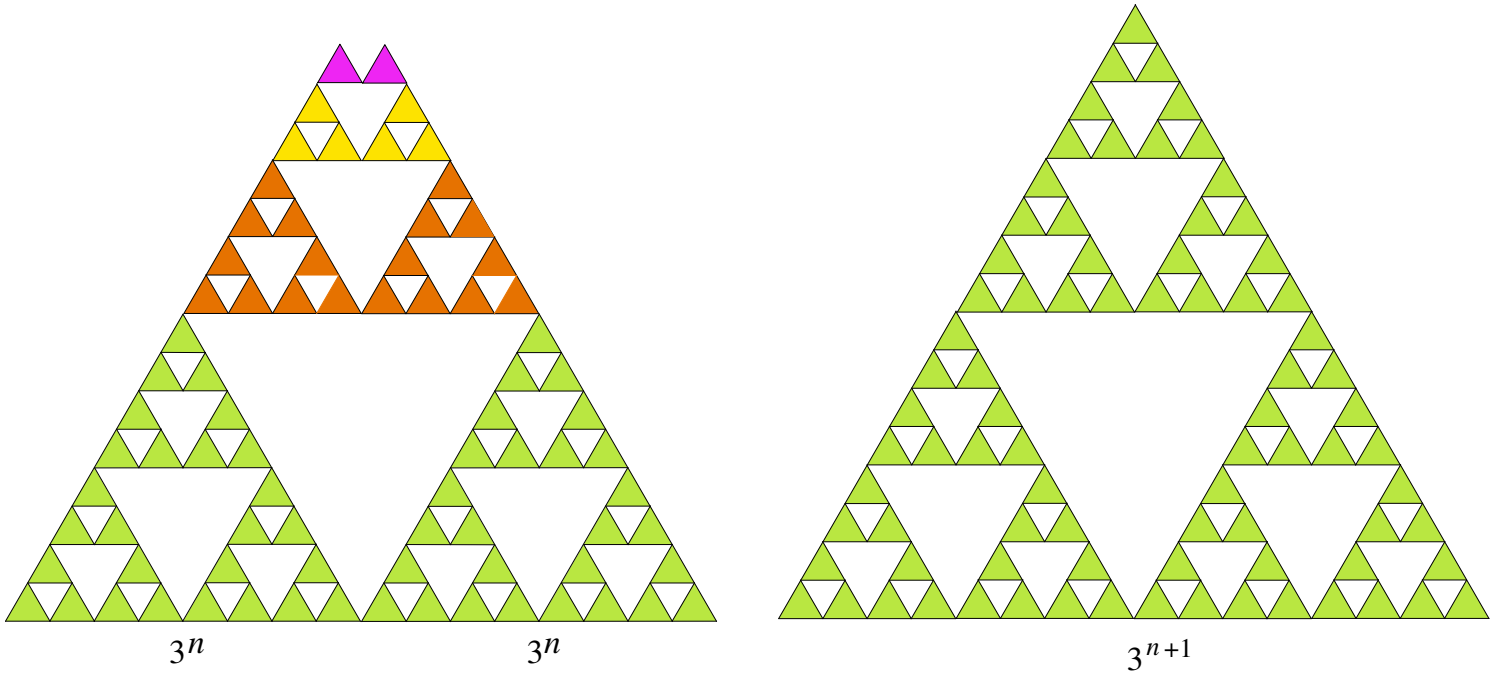

1
3
3^2
3^3
3^n
3^n
3^{n+1}



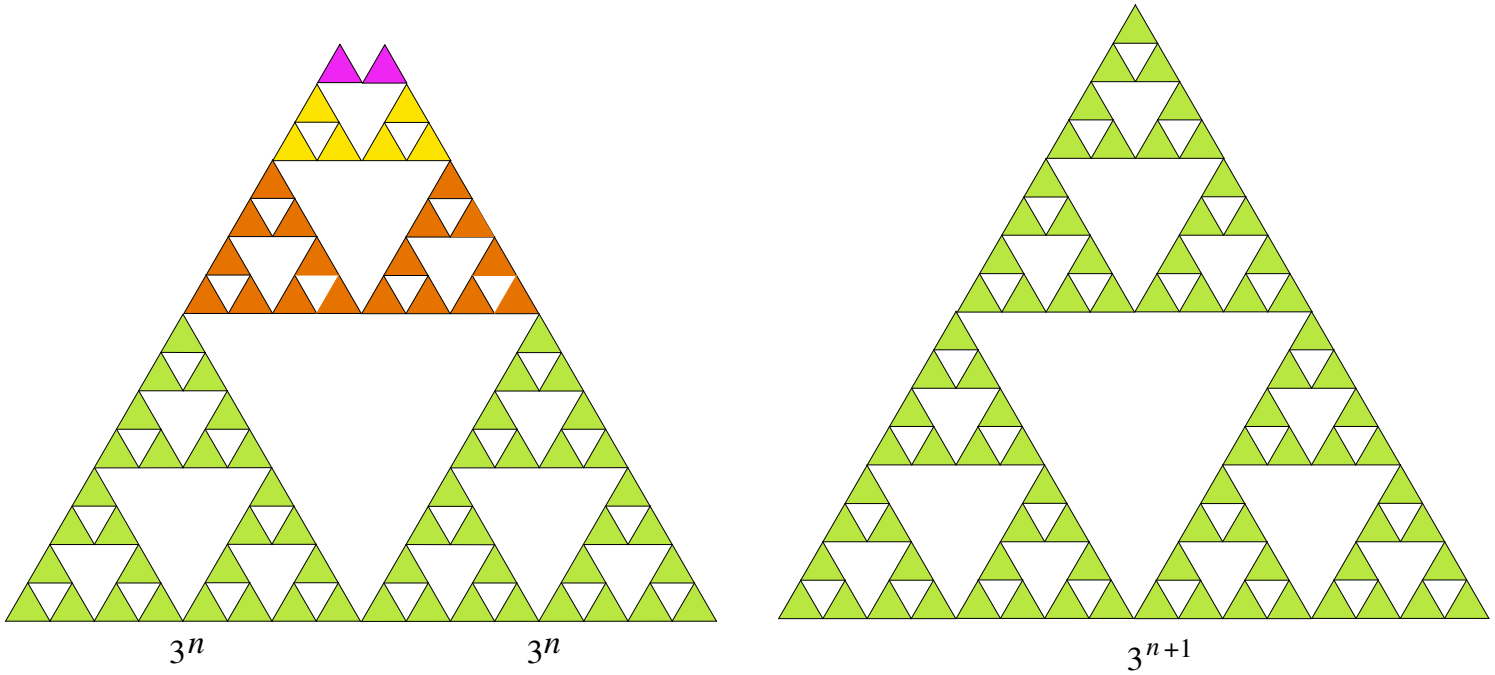
3^n 3^n



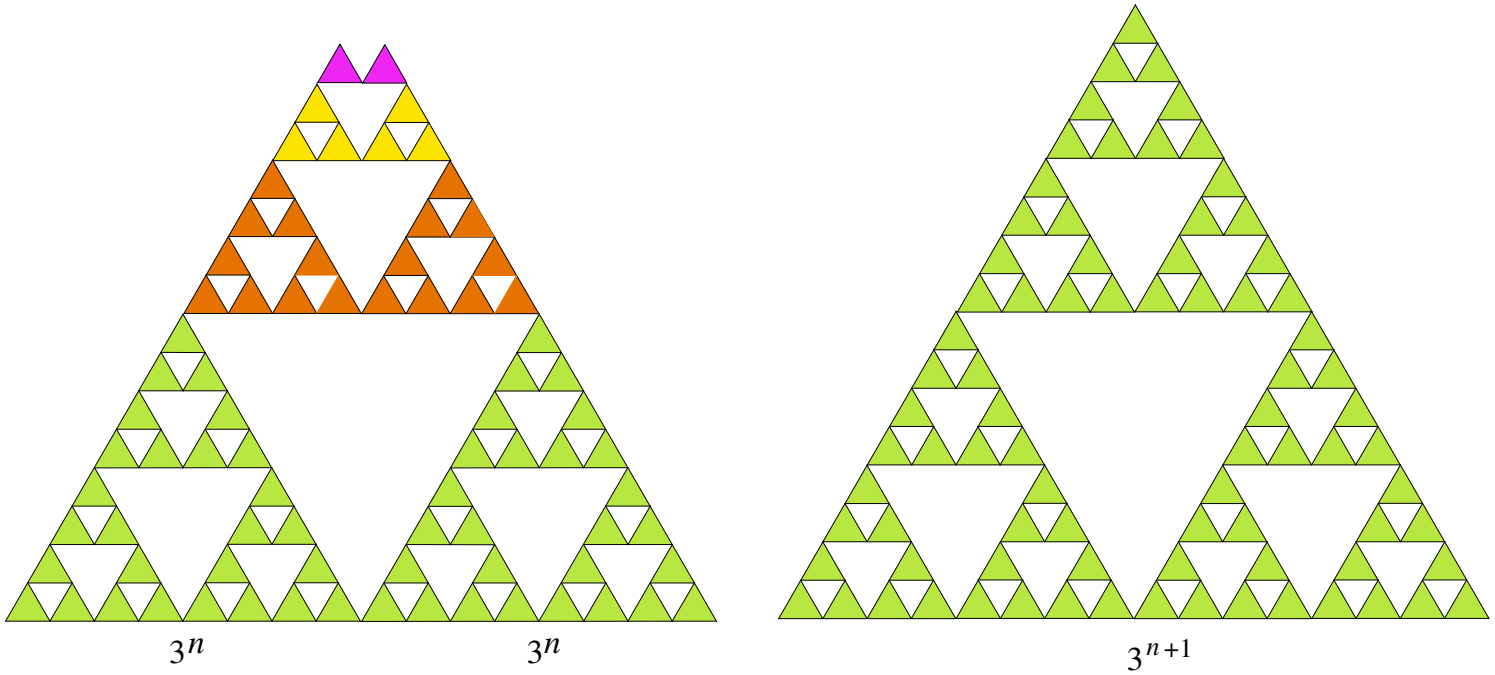
3^{n+1}



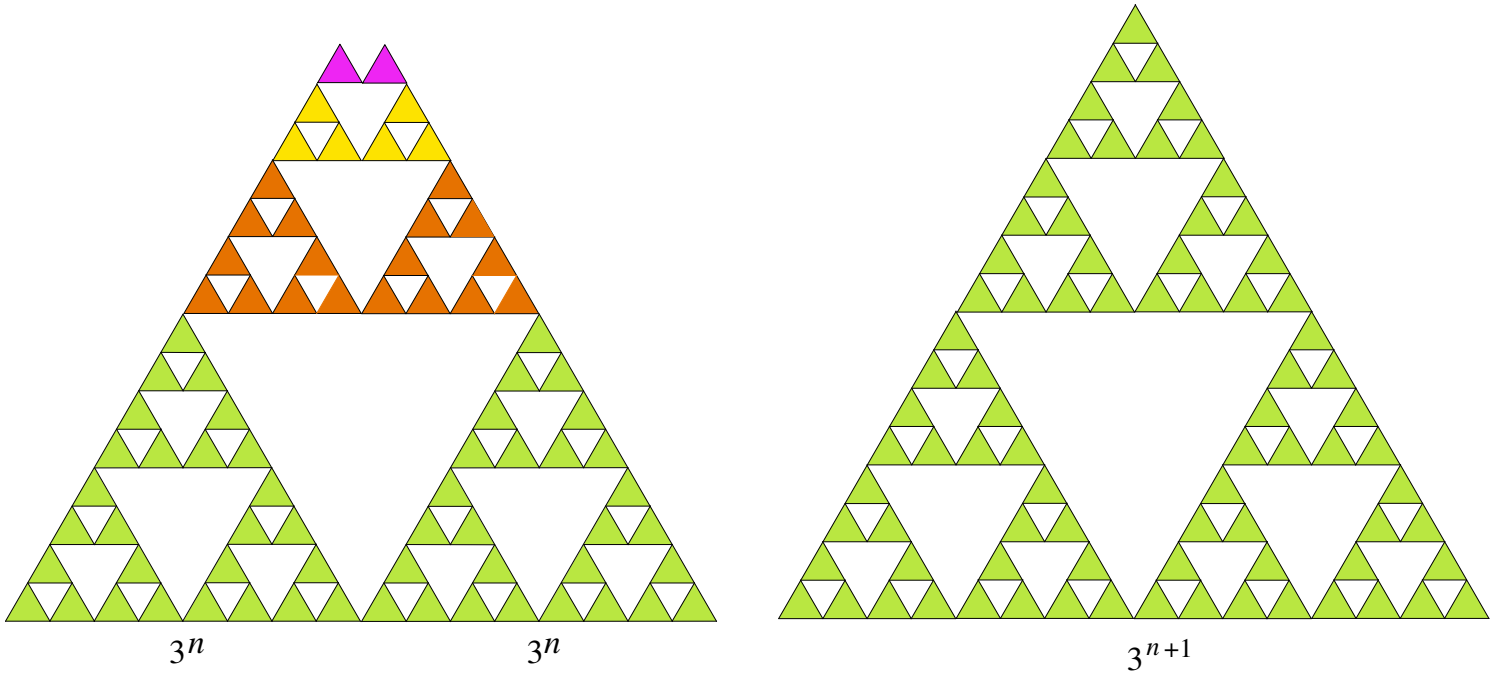
$$2 \cdot 3^n + \cdots + 2 \cdot 3^3 + 2 \cdot 3^2 + 2 \cdot 3 + 2 \cdot 1 =$$



$$2 \cdot 3^n + \cdots + 2 \cdot 3^3 + 2 \cdot 3^2 + 2 \cdot 3 + 2 \cdot 1 = 3^{n+1}$$



$$2 \cdot 3^n + \cdots + 2 \cdot 3^3 + 2 \cdot 3^2 + 2 \cdot 3 + 2 \cdot 1 = 3^{n+1} - 1$$

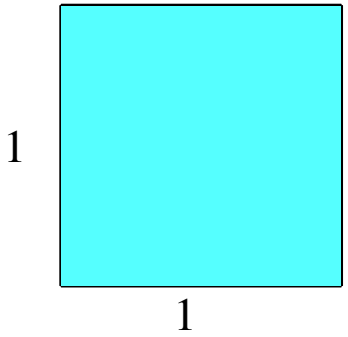


$$2 \cdot 3^n + \cdots + 2 \cdot 3^3 + 2 \cdot 3^2 + 2 \cdot 3 + 2 \cdot 1 = 3^{n+1} - 1$$

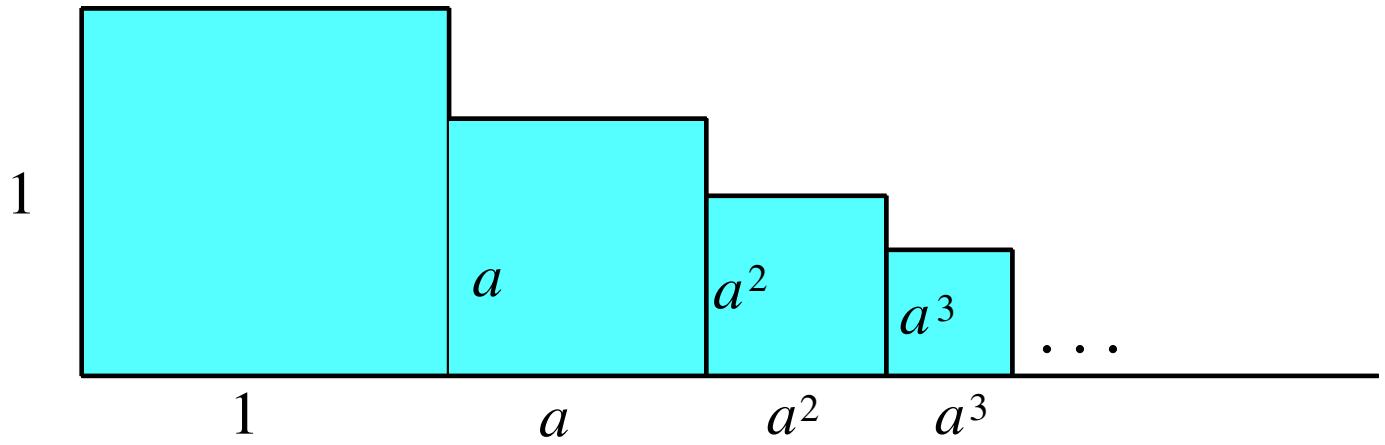
$$1 + 3 + 3^2 + \cdots + 3^n = \frac{3^{n+1} - 1}{2}$$

Nekonečná řada

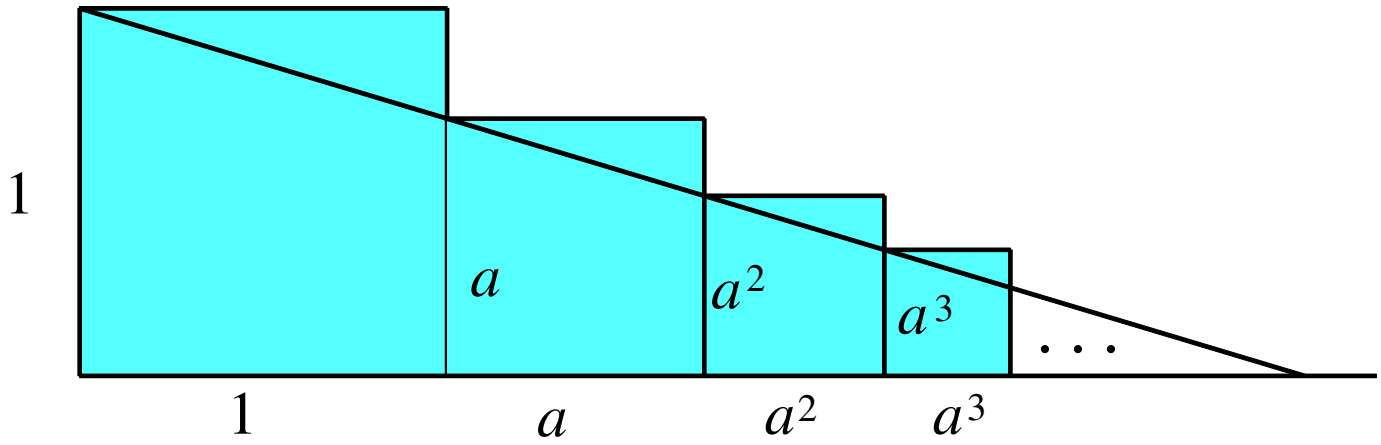
Pro reálné číslo a, $0 < a < 1$, platí $1 + a + a^2 + a^3 + \cdots = \frac{1}{1-a}$.



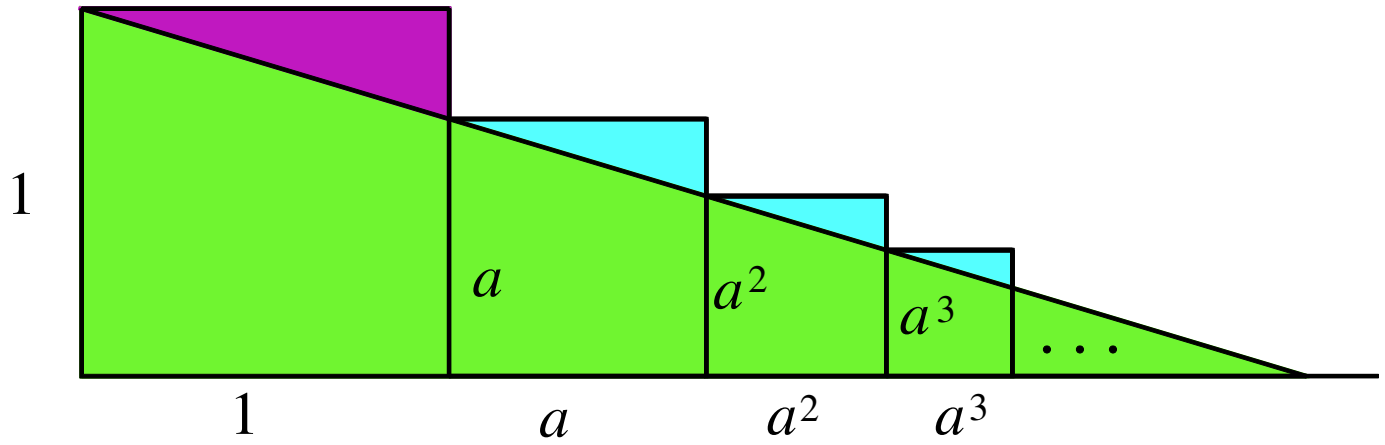
Pro reálné číslo a, $0 < a < 1$, platí $1 + a + a^2 + a^3 + \cdots = \frac{1}{1-a}$.



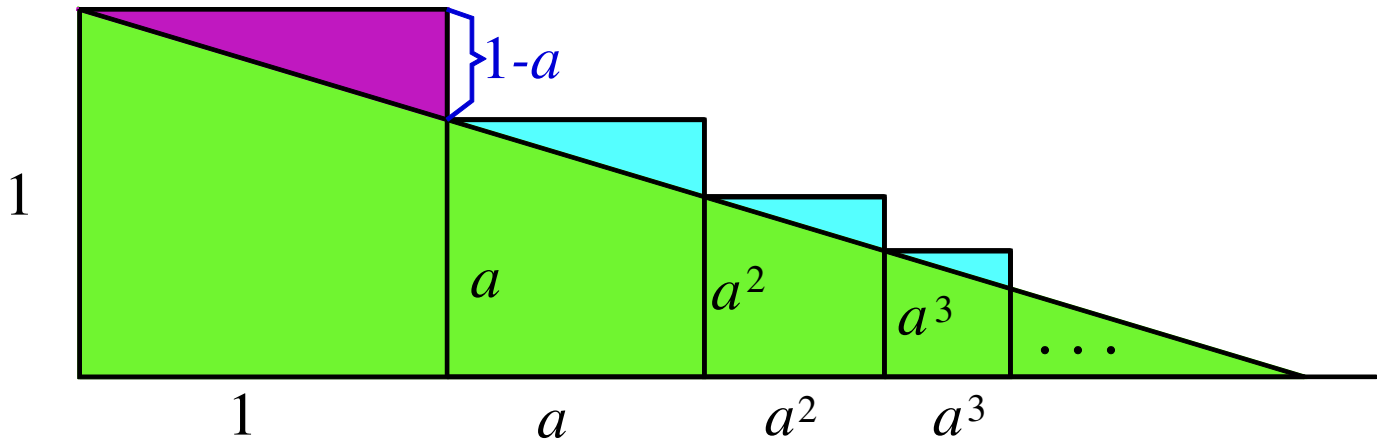
Pro reálné číslo a, $0 < a < 1$, platí $1 + a + a^2 + a^3 + \cdots = \frac{1}{1-a}$.



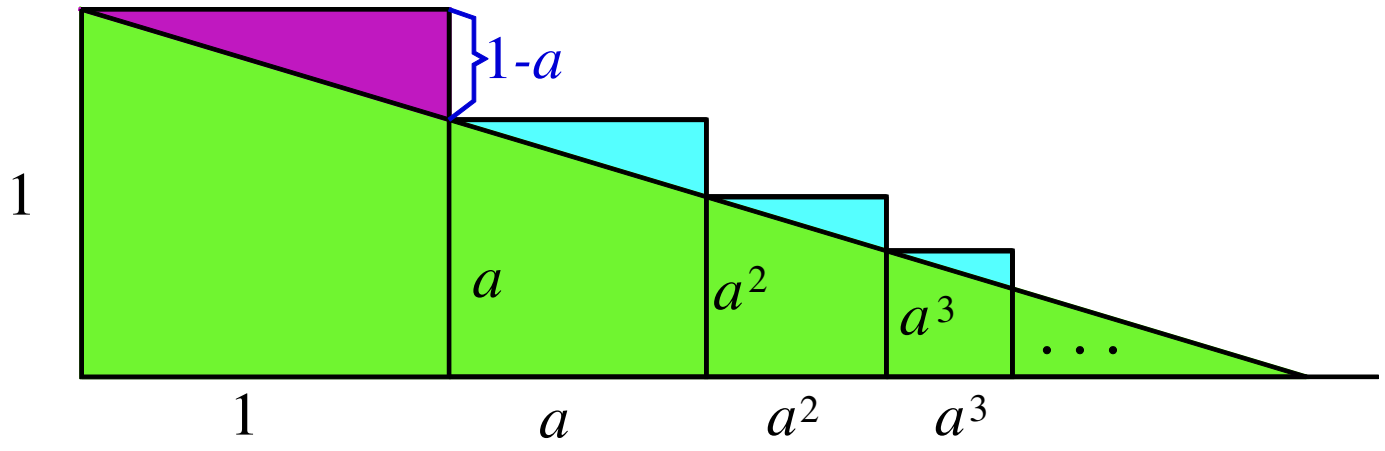
Pro reálné číslo a, $0 < a < 1$, platí $1 + a + a^2 + a^3 + \cdots = \frac{1}{1-a}$.



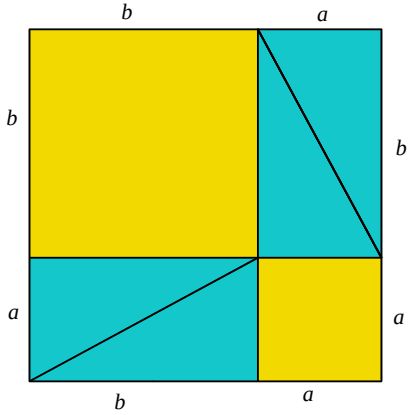
Pro reálné číslo a, $0 < a < 1$, platí $1 + a + a^2 + a^3 + \cdots = \frac{1}{1-a}$.

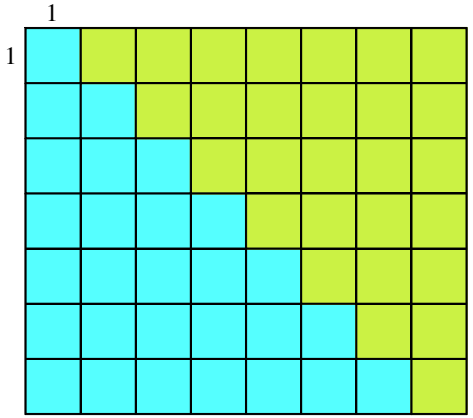


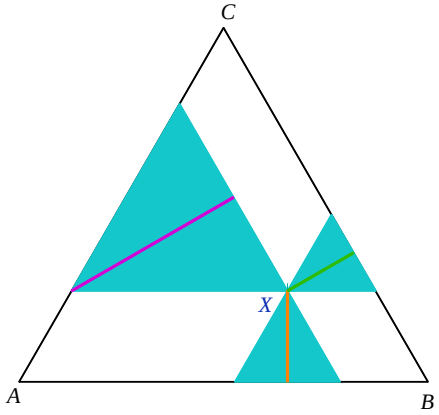
Pro reálné číslo a, $0 < a < 1$, platí $1 + a + a^2 + a^3 + \cdots = \frac{1}{1-a}$.



$$\frac{1}{1-a} = \frac{1 + a + a^2 + a^3 + \cdots}{1} = 1 + a + a^2 + a^3 + \cdots$$







Děkuji Vám za pozornost.