Four-Dimensional Visual Exploration of the Complex Number Plane

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Abstract. A circle and a straight line have two, one, or no intersection point in the real Euclidean plane, because analytically computed, the discriminant is negative. Calculated in R, the given equation has no solution, but in C a solution exists. This solution in C we are unable to visualize in the Euclidean plane. Let us focus first only on the straight line, which we can vizualize in Euclidean plane. If we have the same line described analytically, we can calculate its complex solution.The real Euclidean plane is then not enough to visualize the line, because the complex part of the line lies somewhere "above" or "below" the real Euclidean plane. Therefore, to visualize this staight line, including its complex parts, we need at least a 4-dimensional space. In the talk, we use double orthogonal projection onto two mutually perpendicular 3-spaces (4DDOP) to visualize the 4-dimensional space. In this talk, we vizualize the circle and the line including its complex parts in 4DDOP, with the analytic calculation to support it. Such a representation is useful for a better understanding of the complex number plane and for the ability to not only visualize objects in the complex number plane, but also the ability to draw varius construction in the complex numer plane.

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